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THE SPARE PARTS ALLOCATION PROBLEM

WITH IMPERFECT INFORMATION, I.

by

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ABSTRACT:

Specific decision rules for the spare parts allocation procedure for a system composed of two failure prone items are formulated and analysed. It is considered desirable to have the system operate successfully during a mission when space for only one spare is available. The decision rules formulated consider the use of operational data to decide where a spare is best placed. These rules have been numerically evaluated for several specific exponential time to failure distributions and tables associated with this evaluation are included.

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1. The Problem

Consider a complex system composed of a number of failure prone items (components). In order to enable the system to operate successfully during a mission or patrol it may be desirable to stock a certain number of spare parts for each component. Usually, however, the space and budget available for storing and procuring spares is limited, and a choice must be made between the potential recipients of spares. If, say, Component i is allotted a spare, then it may be necessary to deprive Component j of one (or more). In this paper we wish to report on some preliminary formulations of this problem and their solutions, and to point the way for further research. We deal only with a very simplified setup for the present, but will go on to extend it later.

We emphasize that the problems considered here are extensions of a considerable volume of previous work that typically assumes perfect knowledge of the failure laws of the components and spares. Here we are considering the issues that arise when operational data is used to decide where a spare is best placed; the problem is one of statistical decision theory, and requires an attempt to predict in the face of uncertainty. The emphasis is upon a consideration of certain <u>simple rules</u>, and a numerical evaluation of their performance. Since many simple rules may be considered the present study merely initiates research into this topic. Further reports will continue these investigations.

2. The Case of Two Components and One Spare

Consider the following simple situation. There are two components, and room for one spare. The question: to which component should the spare be allocated? We are asked to choose between the following configurations.

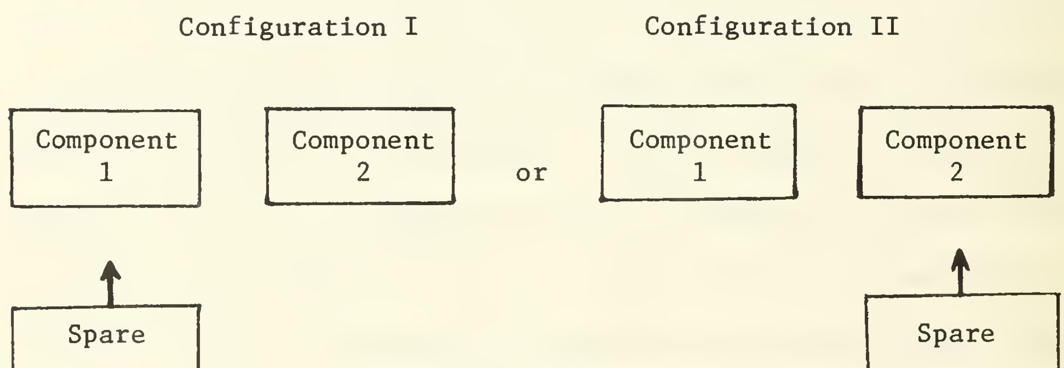

Figure 1-1

We shall do so initially on the basis of the following basic assumptions, any of which may be altered if there is evidence for the need.

Assumptions

(1) The system is required to operate for a fixed mission or patrol time, M. If either component cannot function during the mission there is a penalty or cost. The function of each component is equally important for achieving mission success. This latter assumption will be altered later to bring in relative essentiality of the components.

(2) Both components, and the spare, are fully operational at the beginning of the mission.

(3) Both components are independently subject to failure during the mission. This neglects the possibility of "common cause" failure mechanisms, e.g. as the result of severe shock.

(4) A failed component is immediately replaced by a spare if one is available.

(5) At the time of replacement the spare is operational, i.e. is "as good as new." No breakage or damage during replacement is presumed to occur.

(6) Once the spare is allocated (at mission beginning) the allocation is fixed for that mission. No new spares are available, and repair is impossible. Again, variations can be made as the study progresses.

(7) We are not studying the situation in which the single spare will fit either one of the two failed components. The spare will perform only the function of the component that it is assigned to replace.

3. Spare Allocation To Maximize The Probability That Both Components Function: The Case of Perfect Information.

The problem is to allocate the single spare to Component 1 or to Component 2 at the beginning of a mission of duration M. In terms of Figure 1, it is to choose between Configuration I and Configuration II. There are many possible bases for the choice; we shall initially select that configuration or allocation that <u>maximizes the probability that neither component's function is lost during M</u>. Putting this more abstractly, let T_i denote the time to failure of Component i $(i=1,2)$, and S_i $(i=1,2)$ the time to failure for the spare, if one is assigned. Then if Component 1 is assigned the spare the <u>system</u> (consisting of Component 1 and Component 2) survives M if

$$T_1 + S_1 > M \qquad \text{and} \qquad T_2 > M \tag{3.1}$$

while if Component 2 is given the spare the system survives with probability

$$T_1 > M \qquad \text{and} \qquad T_2 + S_2 > M. \tag{3.2}$$

By virtue of our assumptions we wish to select Configuration 1 (spare Component 1) if

$$P\{T_1 + S_1 > M\}P\{T_2 > M\} > P\{T_1 > M\}P\{T_2 + S_2 > M\}. \tag{3.3}$$

and otherwise to select Configuration 2 (spare Component 2). Actually, the case of equality is a toss-up, and may be resolved on other grounds.

Now if perfect information is available concerning the distributions of the various random variables we can express our decision rules in terms of these distributions. That is, (3.3) becomes

$$\left[1-F_{T_1} * F_{S_1}(M)\right]\left[1-F_{T_2}(M)\right] > \left[1-F_{T_1}(M)\right]\left[1-F_{T_2} * F_{S_2}(M)\right]. \qquad (3.4)$$

where $*$ represents convolution, F_{T_i} is the distribution function (d.f.) of the lifetime of the original Component i, and F_{S_i} is the d.f. of the original spare.

EXAMPLE 3.1. Exponential Distributions.

Suppose that

$$\begin{aligned} F_{T_1}(x) = F_{S_1}(x) &= 1-e^{-\lambda_1 x} \qquad && \lambda_1>0, \quad x\geq 0, \\ &= 0 && x < 0 \end{aligned}$$

and (3.5)

$$\begin{aligned} F_{T_2}(x) = F_{S_2}(x) &= 1-e^{-\lambda_2 x} \qquad && \lambda_2>0, \quad x\geq 0, \\ &= 0 && x < 0 \end{aligned}$$

It is now easy to compute (3.4); since

$$\begin{aligned} 1-F_{T_1} * F_{S_1}(M) &= \int_0^M e^{-\lambda_1(M-x)} e^{-\lambda_1 x} \lambda_1 dx + e^{-\lambda_1 M} \\ &= e^{-\lambda_1 M}\left[1+\lambda_1 M\right] \qquad (3.4) \end{aligned}$$

and thus, by symmetry,

$$1-F_{T_2} * F_{S_2}(M)= e^{-\lambda_2 M}\left[1 + \lambda_2 M\right] , \tag{3.5}$$

so we would decide as follows:

<u>Pick Configuration I if</u> $\quad e^{-\lambda_1 M}\left[1+\lambda_1 M\right]e^{-\lambda_2 M} > e^{-\lambda_2 M}\left[1+\lambda_2 M\right]e^{-\lambda_1 M};$

$$\text{or, } \underline{\text{equivalently}}\text{, if} \qquad \lambda_1 > \lambda_2 \tag{3.6}$$

The latter is an obvious decision rule under the circumstances.

4. <u>Imperfect Information</u>; <u>Exponential Failure Times</u>.

In order to be useful, the simple results of the previous section must be extended to deal with unknown distributions. We shall now discuss this problem in the context of exponential distribution for times to failure. We shall numerically evaluate certain decision rules that depend upon the use of operational data.

4.1 Maximum Likelihood Estimates

The method of maximum likelihood has many appealing qualities, especially for large samples; see Cramér [1]. It is a natural starting point for an excursion into the realm of imperfect information. Methods of Bayesian statistics, to be treated subsequently, are formally quite similar but may advantageously incorporate prior information that likelihood methods cannot handle.

The use of the likelihood (and Bayesian) procedures depend upon the manner in which operational data arises. We consider a few reasonable possibilities.

(1) Continuous operation of failure processes over periods of time t_1 (for Component 1), and t_2 (for Component 2) may be available, most likely from tests. Thus we would, at the time a mission is being planned, have at hand the information that n_1 failures of Component 1 items occurred in t_1, and n_2 of Component 2 items occurred in t_2. Since exponentials govern failure times the likelihood function is

$$L_i(\lambda_i) = e^{-\lambda_i t_i} \frac{(\lambda_i t_i)^{n_i}}{(n_i)!} \tag{3.7}$$

and maximization yields the ordinary maximum likelihood estimate (m.l.e.)

$$\hat{\lambda}_i = \frac{n_i}{t_i} \tag{3.8}$$

According to well-known properties of the m.l.e., the m.l.e. of the probability of system survival under Configuration 1, say, is just

$$e^{-\hat{\lambda}_1 M}\left[1+\hat{\lambda}_1 M\right] e^{-\hat{\lambda}_2 M} \tag{3.9}$$

Under Configuration 2 the m.l.e. of system survival is

$$e^{-\hat{\lambda}_2 M}\left[1+\hat{\lambda}_2 M\right] e^{-\hat{\lambda}_1 M} \tag{3.9,a}$$

With no further information we would prefer Configuration 1 if (3.9a) exceeds (3.9,b). Consequently our m.l.e. decision rule is at first glance, Pick Configuration 1 if $\hat{\lambda}_1 > \hat{\lambda}_2$,

(3.10)

otherwise pick Configuration 2.

Notice that the rule (3.10) must be modified to incorporate the possibility of <u>ties</u>. That is, if $\frac{n_1}{t_1} = \frac{n_2}{t_2}$ we must fall back upon a tie-breaker; for this we shall adopt simple randomization and effectively flip a coin. In practice one could invoke, or search for, other information upon which to base the decision.

Our rule then may be expressed as follows

<u>Decision Rule 1</u>: If $\frac{n_1}{t_1} > \frac{n_2}{t_2}$ choose Configuration 1.
If $\frac{n_1}{t_1} = \frac{n_2}{t_2}$ resolve the tie by effectively flipping a fair coin (if Heads, pick Configuration 1, if Tails, pick Configuration 2).

In Figures 8-1 and 8-8 it can be seen that for certain parameter values, the quality of the decision changes very slightly with increases in the magnitude of experience (length of t_1 and t_2). Ironically, most of the decisions must be made by coin flip with the parameter values selected for study. The probability of a correct decision is never over about 65%, but nevertheless the reliability is improved on the average.

System reliability, using Decision Rule 1, is computed as follows. We compute, numerically, the probability of choosing Configuration 1 under Decision Rule 1. This involves computing

$$p_1 = P\{\frac{n_1}{t_1} > \frac{n_2}{t_2}\} + \frac{1}{2} P\{\frac{n_1}{t_1} = \frac{n_2}{t_2}\} \qquad (3.11)$$

If $\lambda_1 > \lambda_2$ then (3.11) supplies the probability of making a correct decision.

Then the system reliability from following this rule is

$$\underline{R}_1 = p_1 R_1 + (1-p_1)R_2$$

where (3.12)

$$R_1 = e^{-\lambda_1 M}(1+\lambda_1 M)e^{-\lambda_2 M}$$

and

$$R_2 = e^{-\lambda_2 M}(1+\lambda_2 M)e^{-\lambda_1 M}$$

We also tabulate the ratio of system unreliability that results from following Decision Rule 1 to the optimum system of unreliability if λ_1 and λ_2 are <u>known</u>, in which case the spare can be allocated perfectly. Finally, we tabulate the difference between the reliability that obtains when the m.l.e. rule is used and that available under perfect information.

Numerical examples that illustrate the consequences of following the above rule are given in the following tables. It is surprising to see that this apparently sensible rule can sometimes actually give results poorer than those of a simple coin flip! For example, suppose t_2=300(5 patrol lengths), and t_1=60(1 patrol length), and $\lambda_1 = 0.0011$, $\lambda_2 = 0.001$. Obviously Configuration 1 (Component 1 has the spare) should be adopted. However, if t_2 much exceeds t_1 the chance that $\frac{n_1}{t_1} > \frac{n_2}{t_2}$ is small: as $t_2 \to \infty$, $\frac{n_2}{t_2} \to \lambda_2$ in probability, while if $t_1 \to 0$, the probability that $n_1 = 0$ approaches unity. Thus the probability <u>approaches</u> one that Configuration 2 will incorrectly be adopted, as reflected in the probability 0.41 (<0.5) in the table. If the test times are nearly equal this effect does not occur.

(2) Records on component failures over previous missions may be available.

(a) Assume that one has a knowledge of whether or not <u>at least one</u> failure of a particular component has occurred during a mission. This is minimal information, for it keeps no track of <u>when</u> failure occurred. This assumption also simplifies analysis, since the probability of <u>a</u> failure does not require knowledge of the presence or absence of a spare. Suppose, then, that ℓ_{10} missions resulted in zero failures at Component 1, ℓ_{11} resulted in one or more failures at Component 1. Similar notation holds for Component 2. The likelihood function for Component 1 is, when mission length = M,

$$L(\lambda_1) = (1-e^{-\lambda_1 M})^{\ell_{11}}(e^{-\lambda_1 M})^{\ell_{10}} \tag{3.13}$$

Maximization by choice of λ_1 gives

$$\hat{\lambda}_1 = \frac{1}{M}\,\ell n\left(\frac{\ell_{10}+\ell_{11}}{\ell_{10}}\right) \tag{3.14}$$

This leads to

<u>Decision Rule 2</u>: If either (equivalently)

(i) $\ell n\left[\frac{\ell_{10}+\ell_{11}}{\ell_{11}}\right] > \ell n\left[\frac{\ell_{20}+\ell_{21}}{\ell_{20}}\right]$, <u>or</u>

(ii) $\frac{\ell_{10}+\ell_{11}}{\ell_{11}} > \frac{\ell_{20}+\ell_{21}}{\ell_{21}}$, <u>or</u> (3.15)

(iii) $\frac{\ell_{20}}{\ell_{21}} < \frac{\ell_{10}}{\ell_{11}}$

choose Configuration 1. If *equality* holds for (i), (ii), or (iii) resolve the tie by flipping a fair coin (if Heads, pick Configuration 1, if Tails, pick Configuration 2).

Suppose we have records on m missions, each of length M. Then $\ell_{10}+\ell_{11} = \ell_{20}+_{21} = m$, and the above rule may be quoted as

Decision Rule 2(a): If $\ell_{10} < \ell_{20}$, pick Configuration 1.
If $\ell_{10} = \ell_{20}$, flip a fair coin.

The Decision Rule 2 is the easiest one possible to understand and apply. Decision Rule 2 makes no use of the *times* at which failures occur during the mission. Clearly its application will lead to a greater number of allocation errors, and hence lower reliability, than will Decision Rule 1. A set of tables is included that displays the probability of correct decision and the reliabilities obtained and for the failure rates shown very little is lost by utilizing Decision Rule 2. Since record keeping is time consuming, and may not be precise, the comparison of reliabilities obtained from Decision Rules 1 and 2 is of interest.

(b) Next we note the possibility of recording time-of-failure data from previous missions, and the use of the latter to make allocation decisions. The decision rule will not be based upon use of the full maximum likelihood approach, but upon a natural extension of the sampling plan of Decision Rule 2.

Assuming that time information is available we may express the likelihood function analogous to (3.13) in the following form:

$$L(\lambda_1) = \left(e^{-\lambda_1 M}\right)^{\ell_{10}} \prod_{j=1}^{\ell_{11}} e^{-\lambda_1 t_{1j}} \lambda_1 \quad , \quad 0 < t_{1j} < M \qquad (3.16)$$

$$= \left(e^{-\lambda_1 M}\right)^{\ell_{10}} \lambda_1^{\ell_{11}} e^{-\lambda_1 t_{1+}}$$

Here t_{1j} is the time of the first failure of Component 1 during the j^{th} mission on which failure occurs, and t_{1+} is the sum of these failure times. Maximization leads us to the estimate

$$\tilde{\lambda}_1 = \frac{\ell_{11}}{t_{1+} + \ell_{10} M} \quad ; \qquad (3.17)$$

the estimate for λ_2 is obtained by replacing ℓ_{11} by ℓ_{21}, t_{1+} by t_{2+}, and ℓ_{10} by ℓ_{20}. The rule is now

<u>Decision Rule 3</u>: If $\tilde{\lambda}_1 > \tilde{\lambda}_2$ pick Configuration 1; otherwise, pick Configuration 2. If $\tilde{\lambda}_1 = \tilde{\lambda}_2$ (or $\ell_{11} = \ell_{21}$) decide by flipping a fair coin.

Decision Rule 3 has not been evaluated numerically, Actually, bounds on its effectiveness can be obtained in terms of Decision Rules 1 and 2. Suppose we consider Decision Rule 1 applied when experience with Component 1 has extended to m_1 missions, and that for Component 2 is m_2 missions. The experience consists merely of information as to whether the particular component has failed (i) once, or (ii) at least once. Clearly this is less information than is assumed in the derivation of Decision Rule 1, so if we set $t_1 = m_1M$, and $t_2 = m_2M$ the optimum reliability obtained by following Decision Rule 3 will be less than that obtained by following Decision Rule 1. On the other hand, the result of following Decision Rule 3 will be a higher reliability than that from following Decision Rule 2 with $m_1 = \ell_{10}+\ell_{11}$, and $m_2 = \ell_{20}+\ell_{21}$. For small values of λ_1 and λ_2 the bounds should be close, and the evaluation of Rule 3 can thus be avoided.

5. Allocation by Means of Baye's Theorem.

One basis for allocating a spare to one of two (or more components) involves the assumption that the failure rates associated with individual components are random variables. Thus the value of λ_1, the failure rate of Component 1 at a particular installation (ship) is a sample from a particular probability distribution, the so-called prior distribution for λ_1, with density $f_{\lambda_1}(x)$. Similarly, the value of λ_2 is drawn from a prior: $f_{\lambda_2}(y)$. It is convenient to assume the values so drawn to be independently selected, although this is not a strict necessity and may not even be plausible.

The model adopted here recognizes that copies of the same basic design will exhibit different failure characteristics at different installations because of environmental variations. Quite concretely, suppose that a large amount of historical failure data is available from each of a large number of installations. Suppose that successive failures at an installation occur with an exponential law, and that rate estimates can be formed for each installation: for example, we might have $\hat{\lambda}_{11}$, $\hat{\lambda}_{12}$, $\hat{\lambda}_{13}, \ldots \hat{\lambda}_{1k}$, the latter being the estimated failure rates for Component 1 at Locations 1,2,...,k. If each estimate is based on a very long string of observations, then each $\hat{\lambda}$ is very close to the true failure rate at the particular location. Moreover, we may find it reasonable to treat the above data (if available) as a sample from a density, $f_{\lambda_1}(x)$. Given sufficient data we could effectively estimate the probability that an installation chosen at random has a failure rate between a and b $(a<b)$ by computing the ratio:

$$\frac{\text{Number of } \hat{\lambda}\text{'s such that } a < \hat{\lambda} < b}{k} = \text{estimate of } \int_a^b f_{\lambda_1}(x)\,dx \equiv \widehat{\int_a^b f_{\lambda_1}(x)\,dx} \tag{5.1}$$

The estimate improves with large k. One would compute this estimate for many intervals (a,b) in order to obtain a smoothed estimate of the prior density.

Lack of data usually prevents us from utilizing the above estimation possibility. One must assume a plausible and convenient functional form for the prior density and use it. Engineering judgment and experience will help in the choice of the prior. However, experience with even a few failures at the particular installation may be used to revise the prior to create a posterior density that tends to concentrate ever more closely (as experience increases) around the true rate prevailing at the particular installation. The posterior is the result of applying Bayes' Theorem. To do this we require the probability that n failures are in fact recorded, given the value of λ_1. Let this be $p(n|\lambda_1=z)$, where z is a possible value for λ_1. Then Bayes' Theorem states that

$$g_{\lambda_1|n}(z) = \frac{f_{\lambda_1}(z)\; p(n|\lambda_1=z)}{\int_0^\infty f_{\lambda_1}(z)\; p(n|\lambda_1=z)} \tag{5.2}$$

is the density function describing the location of parameter λ_1, in the light of, or conditional upon, the information that n failures have occurred. For further details, see Mosteller and Tukey [4], DeGroot [2].

Having a prior distribution and perhaps some data it is now of interest to apply the posterior to our allocation decision. We sketch the rationalle. Let $R_1(\lambda_1,\lambda_2)$ denote the system reliability in the event that Configuration 1 is adopted (Component 1 is allocated the spare) and λ_1 and λ_2 are the true values of the failure rates of Components 1 and 2. Let $R_2(\lambda_1,\lambda_2)$ represent the reliability under Configuration 2. The rule is

Pick Configuration 1 if $R_1(\lambda_1,\lambda_2) > R_2(\lambda_1,\lambda_2)$;

otherwise, Pick Configuration 2.

with a judgemental decision made if the reliabilities are equal. Now if λ_1 and λ_2 are unknown we are led to consider this as a decision problem with appropriate loss function. For more details see a text on statistical decision theory, e.g. DeGroot [2].

Several loss functions suggest themselves as appropriate for the present problem. We investigate first the very simplest.

<u>First Loss Function</u>

<u>Decision</u>	<u>Loss</u>
Pick Configuration 1	$R_2(\lambda_1,\lambda_2) - R_1(\lambda_1,\lambda_2)$
Pick Configuration 2	$R_1(\lambda_1,\lambda_2) - R_2(\lambda_1,\lambda_2)$

Figure 5-1

In everyday language this assignment of a loss states that if we pick Configuration 1 when the unknown rates are λ_1 and λ_2 then the <u>loss</u> is the difference between the reliability obtained by assigning the spare as in Configuration 2, and that from assigning the spare in Configuration 1. Since in the present model

$$R_2(\lambda_1,\lambda_2) - R_1(\lambda_1,\lambda_2) = e^{-\lambda_1 M} e^{-\lambda_2 M}(\lambda_2 - \lambda_2)M$$

the loss is seen to be positive if $\lambda_2 > \lambda_1$, and negative if $\lambda_1 > \lambda_2$ (interpret this as a gain).

Alternative loss functions will be discussed subsequently, but we will first derive the consequences of this one. Note that we wish to pick that Configuration so as to minimize the loss, but to do so requires information concerning the unknown parameter values λ_1 and λ_2. Bayesian principles lead us to consider the loss averaged over the <u>posterior distribution</u> of λ_1 and λ_2; see [2]. Let $g_1(z)$ and $g_2(w)$ be the posterior sensities for λ_1 and λ_2. Now compute the expected loss under these distributions.

<u>Decision</u>	<u>Expected (Bayes) Loss</u>
Pick Configuration 1	$L_B(1) = \iint [R_2(z, w) - R_1(z,w)]g_1(z)g_2(w)dz\,dw$
Pick Configuration 2	$L_B(2) = -L_B(1)$. (5.6)

Clearly the optimal strategy is to adopt Configuration 1 if $L_B(1) < L_B(2) = -L_B(1)$, or if $L_B(1) < 0$. Equivently, the rule is

$$\text{Pick Configuration 1 if} \iint R_1(z,w)g_1(z)g_2(w)dz\,dw > \iint R_2(z,w)g_1(z)g_2(w)dz\,dw. \tag{5.7}$$

Otherwise, pick Configuration 2.

We now illustrate in some particular cases.

6. <u>Bayes Decision Given Continuous Information</u>

Suppose that n_1 failures of Component 1 are observed in time t_1, and n_2 failures of Component 2 are observed in t_2. Let us assume that the prior densities are of gamma form

$$f_{\lambda_1}(z) = e^{-\alpha_1 z}\frac{(\alpha_1 z)^{k_1}}{\Gamma(k_1+1)}\alpha_1 \qquad (\alpha_1>0, k_1>-1)$$

$$f_{\lambda_2}(w) = e^{-\alpha_2 w}\frac{(\alpha_2 w)^{k_2}}{\Gamma(k_2+1)}\alpha_2 \qquad (\alpha_2>0, k_2>-1) \tag{6.1}$$

Such a form is plausible and convenient; by choice of the arbitrary parameters α_1, k_1, and α_2, k_2, most reasonable priors can be approximated. Now the likelihood functions for λ_1 and λ_2 are of the Poisson form:

$$L_1(\lambda_1) = e^{-\lambda_1 t_1} \frac{(\lambda_1 t_1)^{n_1}}{n_1!}$$

$$L_2(\lambda_2) = e^{-\lambda_2 t_2} \frac{(\lambda_2 t_2)^{n_2}}{n_2!} \tag{6.2}$$

This leads to

Consequently the posterior densities are (K_1 is a constant)

$$\begin{aligned} p_{\lambda_1}(z) &= K_1 \quad f_{\lambda_1}(z) L_1(z) \\ &= K_1 \quad e^{-\alpha_1 z} (\alpha_1 z)^{k_1} \alpha_1 \; e^{-t_1 z} (t_1 z)^{n_1} \\ &= e^{-(\alpha_1+t_1)z} \frac{[(\alpha_1+t_1)z]^{n_1+k_1}}{\Gamma(n_1+k_1+1)} (\alpha_1+t_1) \end{aligned} \tag{6.3}$$

which is again of gamma form. An exactly similar result holds for $p_{\lambda_2}(w)$.

Now it is easy to apply the above distributions to find the relevant allocations. We find (abbreviating by putting $A_i = \alpha_i + t_i$, $B_i = n_i + k_i$):

$$\begin{aligned} &\iint R_1(z,w) p_{\lambda_1}(z) p_{\lambda_2}(w) dz\, dw \\ &= \int_0^\infty\!\!\int_0^\infty e^{-zM}(1+zM) e^{-wM} e^{-A_1 z} \frac{(A_1 z)^{B_1}}{\Gamma(B_1+1)} A_1 dz \; e^{-A_2 w} \frac{(A_2 w)^{B_2}}{\Gamma(B_2+1)} B_2 \, dw. \\ &= \left(\frac{A_2}{A_2+M}\right)^{B_2+1} \left[\left(\frac{A_1}{A_1+M}\right)^{B_1+1} + \frac{M(B_1+1)}{(A_1+M)^{B_1+2}} A_1^{B_1+1}\right] \end{aligned} \tag{6.4}$$

$$\underline{\text{Decision Rule 4:}}\quad \underline{\text{If}}\quad \frac{B_1 + 1}{A_1 + M} > \frac{B_2 + 1}{A_2 + M}$$

$$\underline{\text{or}}\ \underline{\text{if}}\quad \frac{n_1 + k_1 + 1}{\alpha_1 + t_1 + M} > \frac{n_2 + k_2 + 1}{\alpha_2 + t_2 + M} \tag{6.5}$$

Pick Configuration 1;

Otherwise, pick Configuration 2. If a tie occurs, flip a fair coin.

Notice the similarity of the above Bayes rule to Rule 1, the latter being based upon a maximum likelihood estimation. If t_1 and t_2 both become large as compared to α_1, α_2, k_1, k_2, and M, i.e. if considerable experience is available, then Rules 1 and 4 tend to give the same results. However, Rule 4 enjoys the advantage if appropriate values for the prior parameters (α_i, and k_i) are available. The work of Haber and Sitgreaves [3] suggests one way in which prior information may be introduced. In the present study we have not made use of historically de i ed prior information. Instead, we have assumed several possible priors and illustrated their effect upon the probability of making the correct decision (e.g. choosing Configuration 1 if $\lambda_1 > \lambda_2$). The numerical details are discussed in a later section. It will be seen that certain rules, based upon plausible priors, can substantially improve the probability of making the correct decision, and can also improve overall reliability. The Bayesian approach deserves further attention.

7. Play-the-Loser Rules for Spares Allocation; the One-Stage Case

The previous rules for allocation of a single spare to one of two components were derived from rather simple statistical principles. In the present section we discuss an even simpler class of rules that has intuitive appeal and is fairly easy to apply. We call these *play the loser rules*.

The general idea is as follows. Imagine that Configuration 1 is in effect (spare allocated to Component 1, no spare to Component 2) at the beginning of a mission. Let us examine the possible events that may occur during the mission, and ask ourselves which of these might suggest the wisdom of transferring to Configuration 2 for the next mission. The possibilities are the following (we use the symbol "F" to denote a failure, and "$\not{F}$" to denote no failure.

	Component 1		Component 2
1.	$\not{F}$		$\not{F}$
2.	$\not{F}$		F
3.	F	$\not{F}$	$\not{F}$
4.	F	$\not{F}$	F
5.	F	F	$\not{F}$
6.	F	F	F

Offhand, there is no reason to make a change unless Component 2 fails; if this event occurs in concert with at most one failure of Component 1 there is reason to consider Component 2 the more failure prone ("the loser") and hence our rule is:

> Decision Rule 5: If the spare is assigned to Component 1 on a mission switch the spare to Component 2 on the next mission if and only if Cases 2 or 4 occur.

We shall work out the implications of this rule for the case of constant failure rates, after pointing out that it is obviously quite adaptive. If, say, the failure rate of Component 2 changes for the worse, our rule will automatically allocate the spare to Component 2 with higher probability. Of course, an occasional freak outcome will send the spare back to Component 1, but this will happen infrequently. Various refinements of this rule are possible, and are under investigation.

Analysis of the One-Stage Play-the-Loser Rule.

Because of our assumption that failures occur independently and exponentially the imposition of the rule means that the spare location is a simple Markov chain with stationary transition probabilities. Let $U_1(t)$ denote the probability that Configuration 1 is in force at the beginning of patrol or mission t, and $U_2(t)$ be the probability of Configuration 2. The one-step transition probability matrix clearly appears as below.

Config. at t \ Config. at t+1	1	2
1	$1 - p_{12} = p_{11}$	$p_{12} = e^{-\lambda_1 M}(1+\lambda_1 M)(1-e^{-\lambda_2 M})$
2	$p_{21} = e^{-\lambda_2 M}(1+\lambda_2 M)(1-e^{-\lambda_1 M})$	$1 - p_{21} = p_{22}$

The usual conditional probability arguments relating to Markov chains now show that

$$U_1(t+1) = U_1(t)p_{11} + U_2(t)p_{21}. \tag{7.1}$$

Since the chain is irreducible and ergodic there is a stationary or long-run distribution denoted by $U_i (i=1, 2)$,

$$U_i = \lim_{t\to\infty} U_i(t); \tag{7.2}$$

since $U_2 = 1 - U_1$ we get immediately from (7.1) the information that

$$U_1 = \frac{p_{21}}{p_{21} + p_{12}} \tag{7.3}$$

Now the system reliability, given Configuration 1, is

$$R_1 = e^{-\lambda_1 M}(1+\lambda_1 M)e^{-\lambda_2 M} \tag{7.4}$$

while <u>given</u> Configuration 2 it is

$$R_2 = e^{-\lambda_2 M}(1+\lambda_2 M)e^{-\lambda_1 M} \tag{7.5}$$

Hence the long-run system reliability obtained by following our one-stage play-the-loser rule is

$$R = U_1\ T_1 + U_2\ R_2 \tag{7.6}$$

We present later a tabulation illustrating the result of applying the one-stage rule.

8. Numerical Illustrations

In this section we present the results of applying some of our decision rules, given varying amounts of information. The amount of information is represented in terms of the length of previous patrols or missions over which we have information available. A basic patrol length is taken to be 60 days. We have chosen to tabulate (a) the probability of correct decision under each of several rules, and (b) the ratio of system unreliability, using the rule in question, to system unreliability given perfect information about the rates. Perhaps most informatively, we have tabulated (c) the difference in the probability of making a correct decision, using various rules, and the probability of correct decision, using the maximum likelihood rule. Our procedure of comparing the probability of a correct decision or allocation under our rules with that derived from maximum likelihood is arbitrary, and can easily be altered if desired.

Specific Rules

The particular rules considered are the following:

Rule 1. This is the maximum likelihood estimate procedure discussed first, in Section 3.

Rule 2. This is the rule that uses only the information that at least one failure had occurred during a mission, and does not refer to the timing of that failure.

Rule 4,A. This is the Bayes-Derived rule, (6.5), with $\alpha_i = k_i = 0$, i.e. a very diffuse exponential prior distribution that is the same for both Component 1 and Component 2.

Rule 4,B. Bayes rule with the (improper) prior having $k_i = -1$, $\alpha_i = 0$, i.e. a diffuse prior with relatively high mass near $\lambda = 0$.

Rule 4,C. Bayes rule with mean at about $\lambda = 0.001$ and a moderate spread around that value.

Discussion of the Numerical Tablulations

1. The numerical values computed do not show that any of the rules for allocation are "best" over all parameter values introduced. Other rules are in the process of development, and their performance can be numerically considered as well.
2. Here we summarize the behavior of our rules for the test cases considered. The numbers suggest broader generalizations and further investigations; the latter are under way.

First Case: $\lambda_2 = 0.001$, $\lambda_1 = 0.0012\ (0.001)\ 0.0022$.

Rule 1 (Maximum Likelihood) performs better than the other rules, with the occasional exception of Rule 4,C. Rule 4,C performs better than Rule 1 when $t_2 > t_1$, and less well when $t_1 > t_2$. The reason for this remains to be given. The relatively poor performance of Rule 4,A is probably the result of concentrating too much prior probability at high λ-values.

Second Case: $\lambda_2 = 0.01$, $\lambda_1 = 0.001\ (0.001)\ 0.011$.

Rule 4,A now betters Rule 1 and the other rules rather consistently. Possibly this is because the diffuse exponential prior more properly represents the λ_2 value chosen here.

Comment: Rule 2 has also been compared to Rule 1. In the first case above, very little difference in the behavior of Rules 1 and 2 is noted. This is because of the small failure rates (λ-values) under comparison. There is only a small chance that either component fails during a patrol, and the extra information supplied by the timing of a failure is nearly insignificant. The above fact has important implications for record keeping and data collection systems. It means that errors in recording the precise timing of failures makes almost no difference when failure rates are small. Just the fact that failure occurs during a patrol is enough to guide the decision maker to reasonable allocations.

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F(N2)=N2*T1/T2

TABLE OF VALUES FOR P(CORRECT DECISION) USING M.L.E.DECISION RULE

LAMBDA2 = 0.00100
M = 60.00000

LAMBDA1 VALUES

T2	T1	0.0012	0.0013	0.0014	0.0015	0.0016	0.0017	0.0018	0.0019	0.0020	0.0021	0.0022
60.	60.	0.505628	0.508419	0.511194	0.513953	0.516697	0.519425	0.522138	0.524835	0.527518	0.530185	0.532837
60.	120.	0.534312	0.539223	0.544079	0.548881	0.553630	0.558326	0.562970	0.567562	0.572104	0.576596	0.581038
60.	180.	0.562400	0.569180	0.575840	0.582383	0.588812	0.595128	0.601334	0.607432	0.613424	0.619311	0.625096
60.	240.	0.588722	0.597397	0.605273	0.613257	0.621052	0.628663	0.636095	0.643351	0.650437	0.657356	0.664112
60.	300.	0.613242	0.622952	0.632375	0.641520	0.650394	0.659007	0.667366	0.675478	0.683351	0.690992	0.698407
120.	60.	0.481891	0.484971	0.488033	0.491076	0.494102	0.497110	0.500099	0.503071	0.506025	0.508962	0.511881
120.	120.	0.510603	0.515821	0.520983	0.526091	0.531145	0.536145	0.541092	0.545987	0.550830	0.555621	0.560362
120.	180.	0.531766	0.538486	0.545112	0.551648	0.558093	0.564449	0.570717	0.576899	0.582994	0.589005	0.594933
120.	240.	0.556428	0.564640	0.572685	0.580570	0.588295	0.595867	0.603287	0.610560	0.617688	0.624675	0.631524
120.	300.	0.578162	0.587460	0.596509	0.605316	0.613890	0.622238	0.630369	0.638289	0.646006	0.653525	0.660854
180.	60.	0.458062	0.461311	0.464521	0.467722	0.470903	0.474066	0.477210	0.480335	0.483441	0.486529	0.489598
180.	120.	0.493945	0.499828	0.505648	0.511400	0.517084	0.522703	0.528256	0.533745	0.539170	0.544532	0.549831
180.	180.	0.515038	0.522389	0.529630	0.536763	0.543789	0.550710	0.557527	0.564243	0.570858	0.577374	0.583792
180.	240.	0.527345	0.535583	0.543690	0.551668	0.559519	0.567244	0.574844	0.582322	0.589679	0.596917	0.604037
180.	300.	0.551595	0.561371	0.570949	0.580332	0.589524	0.598528	0.607346	0.615982	0.624440	0.632722	0.640832
240.	60.	0.435456	0.438832	0.442189	0.445526	0.448842	0.452139	0.455416	0.458673	0.461911	0.465130	0.468329
240.	120.	0.471612	0.477662	0.483643	0.489556	0.495401	0.501180	0.506892	0.512539	0.518121	0.523640	0.529095
240.	180.	0.506857	0.515298	0.523594	0.531748	0.539762	0.547638	0.555380	0.562989	0.570468	0.577818	0.585042
240.	240.	0.519024	0.528269	0.537341	0.546242	0.554976	0.563545	0.571953	0.580203	0.588298	0.596240	0.604032
240.	300.	0.522016	0.531619	0.541056	0.550330	0.559443	0.568396	0.577191	0.585831	0.594316	0.602649	0.610832
300.	60.	0.414145	0.417650	0.421133	0.424596	0.428038	0.431460	0.434861	0.438241	0.441602	0.444942	0.448262
300.	120.	0.454396	0.460867	0.467261	0.473580	0.479823	0.485993	0.492089	0.498114	0.504066	0.509948	0.515760
300.	180.	0.486283	0.494920	0.503412	0.511762	0.519973	0.528045	0.535982	0.543786	0.551460	0.559004	0.566423
300.	240.	0.519868	0.530586	0.541064	0.551308	0.561322	0.571113	0.580684	0.590041	0.599189	0.608132	0.616875
300.	300.	0.522635	0.533579	0.544280	0.554744	0.564976	0.574981	0.584763	0.594327	0.603678	0.612820	0.621758

Fig. 8-1

DECISION RULE 4, A
F(N2)=(N2+1)(T1+M)/(T2+M)-1

TABLE OF VALUES FOR P(CORRECT DECISION) USING DECISION RULE

LAMBDA2 = 0.00100
M = 60.0000

LAMBDA1 VALUES

T2	T1	0.0012	0.0013	0.0014	0.0015	0.0016	0.0017	0.0018	0.0019	0.0020	0.0021	0.0022
60.	60.	0.505628	0.508419	0.511194	0.513953	0.516697	0.519425	0.522138	0.524835	0.527518	0.530185	0.532837
60.	120.	0.126581	0.136355	0.146016	0.155567	0.165007	0.174339	0.183563	0.192681	0.201695	0.210605	0.219413
60.	180.	0.101042	0.109344	0.117620	0.125867	0.134084	0.142271	0.150425	0.158546	0.166632	0.174682	0.182694
60.	240.	0.032316	0.037341	0.042640	0.048197	0.053997	0.060025	0.066268	0.072712	0.079344	0.086151	0.093123
60.	300.	0.026894	0.031217	0.035836	0.040649	0.045735	0.051053	0.056594	0.062345	0.068298	0.074442	0.080767
120.	60.	0.069237	0.074786	0.080302	0.085784	0.091234	0.096652	0.102037	0.107390	0.112711	0.118001	0.123259
120.	120.	0.510603	0.515821	0.520983	0.526091	0.531145	0.536145	0.541092	0.545987	0.550830	0.555621	0.560362
120.	180.	0.174455	0.187548	0.200435	0.213119	0.225604	0.237891	0.249985	0.261887	0.273602	0.285133	0.296481
120.	240.	0.222284	0.238139	0.253632	0.268772	0.283568	0.298030	0.312165	0.325982	0.339489	0.352693	0.365603
120.	300.	0.157101	0.169760	0.182326	0.194794	0.207162	0.219425	0.231579	0.243623	0.255552	0.267365	0.279059
180.	60.	0.139391	0.144537	0.149652	0.154736	0.159790	0.164814	0.169807	0.174771	0.179706	0.184610	0.189486
180.	120.	0.132315	0.142516	0.152598	0.162561	0.172407	0.182137	0.191753	0.201256	0.210647	0.219927	0.229098
180.	180.	0.515038	0.522389	0.529630	0.536763	0.543789	0.550710	0.557527	0.564243	0.570858	0.577374	0.583792
180.	240.	0.214218	0.229882	0.245239	0.260294	0.275054	0.289525	0.303711	0.317619	0.331253	0.344620	0.357723
180.	300.	0.256822	0.274738	0.292187	0.309183	0.325740	0.341869	0.357583	0.372893	0.387811	0.402348	0.416514
240.	60.	0.069465	0.075031	0.080564	0.086064	0.091530	0.096965	0.102366	0.107736	0.113073	0.118378	0.123652
240.	120.	0.133871	0.144183	0.154371	0.164439	0.174387	0.184216	0.193928	0.203524	0.213007	0.222376	0.231634
240.	180.	0.189950	0.204044	0.217893	0.231500	0.244870	0.258007	0.270916	0.283599	0.296062	0.308307	0.320340
240.	240.	0.519024	0.528269	0.537341	0.546242	0.554976	0.563545	0.571953	0.580203	0.588298	0.596240	0.604032
240.	300.	0.247610	0.265323	0.282631	0.299542	0.316067	0.332213	0.347988	0.363402	0.378460	0.393173	0.407547
300.	60.	0.084979	0.090452	0.095893	0.101302	0.106678	0.112022	0.117333	0.122613	0.127862	0.133079	0.138265
300.	120.	0.230091	0.239255	0.248309	0.257256	0.266097	0.274832	0.283463	0.291992	0.300419	0.308746	0.316973
300.	180.	0.190732	0.204879	0.218778	0.232435	0.245853	0.259037	0.271990	0.284716	0.297221	0.309506	0.321577
300.	240.	0.242150	0.259454	0.276361	0.292882	0.309025	0.324798	0.340211	0.355271	0.369986	0.384364	0.398414
300.	300.	0.522635	0.533579	0.544280	0.554744	0.564976	0.574981	0.584763	0.594327	0.603678	0.612820	0.621758

F(N2)=N2(T1+M)/(T2+M)

TABLE OF VALUES FOR P(CORRECT DECISION) USING DECISION RULE

LAMBDA2 = 0.0100
M = 60.0000

LAMBDA1 VALUES

T2	T1	0.0012	0.0013	0.0014	0.0015	0.0016	0.0017	0.0018	0.0019	0.0020	0.0021	0.0022
60.	60.	0.505628	0.508419	0.511194	0.513953	0.516697	0.519425	0.522138	0.524835	0.527518	0.530185	0.532837
60.	120.	0.534566	0.539517	0.544416	0.549264	0.554060	0.558806	0.563502	0.568148	0.572746	0.577295	0.581796
60.	180.	0.562970	0.569840	0.576596	0.583240	0.589775	0.596202	0.602523	0.608740	0.614855	0.620869	0.626785
60.	240.	0.588896	0.597314	0.605539	0.613577	0.621433	0.629110	0.636614	0.643950	0.651120	0.658130	0.664984
60.	300.	0.613424	0.623179	0.632654	0.641858	0.650798	0.659483	0.667921	0.676120	0.684087	0.691829	0.699353
120.	60.	0.481677	0.484740	0.487786	0.490813	0.493823	0.496815	0.499789	0.502746	0.505685	0.508606	0.511511
120.	120.	0.510603	0.515821	0.520983	0.526091	0.531145	0.536145	0.541092	0.545987	0.550830	0.555621	0.560362
120.	180.	0.531771	0.538491	0.545119	0.551656	0.558103	0.564460	0.570730	0.576914	0.583012	0.589025	0.594955
120.	240.	0.558084	0.566536	0.574833	0.582977	0.590970	0.598815	0.606514	0.614070	0.621486	0.628763	0.635905
120.	300.	0.580570	0.590203	0.599596	0.608755	0.617688	0.626400	0.634898	0.643187	0.651273	0.659162	0.666859
180.	60.	0.457581	0.460783	0.463967	0.467132	0.470278	0.473406	0.476515	0.479605	0.482678	0.485732	0.488768
180.	120.	0.493936	0.499824	0.505643	0.511394	0.517078	0.522696	0.528249	0.533737	0.539161	0.544522	0.549820
180.	180.	0.515038	0.522389	0.529630	0.536763	0.543789	0.550710	0.557527	0.564243	0.570858	0.577374	0.583792
180.	240.	0.527345	0.535583	0.543691	0.551669	0.559519	0.567244	0.574844	0.582323	0.589680	0.596918	0.604038
180.	300.	0.551632	0.561416	0.571004	0.580398	0.589601	0.598618	0.607450	0.616101	0.624574	0.632873	0.641000
240.	60.	0.435331	0.438698	0.442045	0.445372	0.448679	0.451967	0.455235	0.458483	0.461712	0.464922	0.468113
240.	120.	0.471612	0.477662	0.483643	0.489556	0.495401	0.501179	0.506891	0.512538	0.518120	0.523639	0.529094
240.	180.	0.506857	0.515298	0.523594	0.531747	0.539762	0.547638	0.555380	0.562989	0.570467	0.577818	0.585042
240.	240.	0.519024	0.528269	0.537341	0.546242	0.554976	0.563545	0.571953	0.580203	0.588298	0.596240	0.604032
240.	300.	0.522016	0.531619	0.541056	0.550330	0.559443	0.568396	0.577191	0.585831	0.594316	0.602649	0.610832
300.	60.	0.414016	0.417511	0.420985	0.424438	0.427871	0.431283	0.434674	0.438046	0.441397	0.444728	0.448040
300.	120.	0.452316	0.458641	0.464893	0.471072	0.477180	0.483218	0.489186	0.495085	0.500916	0.506680	0.512378
300.	180.	0.486251	0.494884	0.503371	0.511716	0.519921	0.527988	0.535919	0.543717	0.551384	0.558923	0.566335
300.	240.	0.519868	0.530586	0.541064	0.551308	0.561322	0.571113	0.580684	0.590041	0.599189	0.608132	0.616875
300.	300.	0.522635	0.533579	0.544280	0.554744	0.564976	0.574981	0.584763	0.594327	0.603678	0.612820	0.621758

Fig. 8-3

DECISION RULE 4C
F(N2)=((N2+.11)(M+T1+111)/(M+T2+111))-.11

TABLE OF VALUES FOR P(CORRECT DECISION) USING DECISION RULE

LAMBDA2 = 0.00100
M = 60.00000

LAMBDA1 VALUES

T2	T1	0.0012	0.0013	0.0014	0.0015	0.0016	0.0017	0.0018	0.0019	0.0020	0.0021	0.0022
60.	60.	0.505628	0.508419	0.511194	0.513953	0.516697	0.519425	0.522138	0.524835	0.527518	0.530185	0.532837
60.	120.	0.126835	0.136650	0.146355	0.155951	0.165439	0.174820	0.184097	0.193269	0.202338	0.211306	0.220173
60.	180.	0.184094	0.197814	0.211302	0.224565	0.237604	0.250425	0.263030	0.275424	0.287609	0.299590	0.311369
60.	240.	0.237604	0.254651	0.271316	0.287609	0.303539	0.319112	0.334338	0.349223	0.363776	0.378004	0.391914
60.	300.	0.285054	0.304554	0.323492	0.341884	0.359746	0.377096	0.393948	0.410317	0.426220	0.441668	0.456677
120.	60.	0.894330	0.894925	0.895517	0.896105	0.896690	0.897272	0.897851	0.898426	0.898998	0.899567	0.900132
120.	120.	0.510623	0.515821	0.520983	0.526091	0.531145	0.536145	0.541092	0.545987	0.550830	0.555621	0.560362
120.	180.	0.174459	0.187554	0.200442	0.213127	0.225613	0.237902	0.249998	0.261903	0.273620	0.285152	0.296503
120.	240.	0.225613	0.241955	0.257955	0.273620	0.288956	0.303971	0.318671	0.333063	0.347152	0.360947	0.374452
120.	300.	0.273585	0.292697	0.311308	0.329431	0.347080	0.364265	0.381000	0.397295	0.413163	0.428615	0.443660
180.	60.	0.845750	0.846593	0.847431	0.848265	0.849094	0.849919	0.850739	0.851555	0.852367	0.853174	0.853977
180.	120.	0.855562	0.857136	0.858692	0.860232	0.861755	0.863262	0.864752	0.866226	0.867684	0.869127	0.870553
180.	180.	0.515038	0.522389	0.529630	0.536763	0.543789	0.550710	0.557527	0.564243	0.570858	0.577374	0.583792
180.	240.	0.214218	0.229882	0.245239	0.260294	0.275055	0.289525	0.303712	0.317620	0.331254	0.344620	0.357724
180.	300.	0.260294	0.278699	0.296653	0.314167	0.331253	0.347919	0.364177	0.380035	0.395505	0.410594	0.425312
240.	60.	0.799803	0.800864	0.801920	0.802970	0.804015	0.805054	0.806087	0.807116	0.808139	0.809156	0.810169
240.	120.	0.812161	0.814146	0.816111	0.818055	0.819978	0.821881	0.823764	0.825627	0.827471	0.829295	0.831100
240.	180.	0.823764	0.826551	0.829295	0.831995	0.834653	0.837270	0.839844	0.842379	0.844873	0.847329	0.849745
240.	240.	0.519024	0.528269	0.537341	0.546242	0.554976	0.563545	0.571953	0.580203	0.588298	0.596240	0.604032
240.	300.	0.247610	0.265323	0.282631	0.299542	0.316067	0.332213	0.347988	0.363402	0.378460	0.393173	0.407547
300.	60.	0.758582	0.760006	0.761422	0.762829	0.764229	0.765619	0.767002	0.768377	0.769743	0.771101	0.772452
300.	120.	0.770971	0.773322	0.775651	0.777956	0.780238	0.782498	0.784735	0.786949	0.789141	0.791311	0.793459
300.	180.	0.784672	0.787975	0.791229	0.794433	0.797588	0.800695	0.803754	0.806768	0.809735	0.812657	0.815535
300.	240.	0.797587	0.801719	0.805767	0.809734	0.813620	0.817427	0.821157	0.824812	0.828392	0.831900	0.835337
300.	300.	0.522635	0.533579	0.544280	0.554744	0.564976	0.574981	0.584763	0.594327	0.603678	0.612820	0.621758

RULE 4A- MLF

DIFFERENCE TABLE FOR P(CORRECT DECISION)

LAMBDA2 = 0.00100
M = 60000.00

LAMBDA1 VALUES

T2	T1	0.0012	0.0013	0.0014	0.0015	0.0016	0.0017	0.0018	0.0019	0.0020	0.0021	0.0022
60.	60.	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
60.	120.	-0.407731	-0.402868	-0.398062	-0.393314	-0.388622	-0.383987	-0.379406	-0.374881	-0.370409	-0.365991	-0.361625
60.	180.	-0.461358	-0.459835	-0.458220	-0.456517	-0.454728	-0.452857	-0.450909	-0.448886	-0.446792	-0.444629	-0.442402
60.	240.	-0.556406	-0.559756	-0.562633	-0.565060	-0.567055	-0.568638	-0.569827	-0.570640	-0.571093	-0.571204	-0.570989
60.	300.	-0.586348	-0.591735	-0.596569	-0.600871	-0.604659	-0.607954	-0.610772	-0.613133	-0.615053	-0.616550	-0.617640
120.	60.	-0.412653	-0.410185	-0.407731	-0.405292	-0.402868	-0.400458	-0.398062	-0.395681	-0.393314	-0.390961	-0.388622
120.	120.	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
120.	180.	-0.357312	-0.350938	-0.344677	-0.338528	-0.332490	-0.326558	-0.320733	-0.315011	-0.309392	-0.303873	-0.298452
120.	240.	-0.334144	-0.326501	-0.319054	-0.311798	-0.304727	-0.297837	-0.291122	-0.284578	-0.278199	-0.271982	-0.265920
120.	300.	-0.421061	-0.417701	-0.414183	-0.410521	-0.406728	-0.402813	-0.398789	-0.394666	-0.390453	-0.386160	-0.381795
180.	60.	-0.318670	-0.316764	-0.314869	-0.312986	-0.311113	-0.309252	-0.307402	-0.305563	-0.303736	-0.301919	-0.300112
180.	120.	-0.361625	-0.357312	-0.353049	-0.348838	-0.344677	-0.340566	-0.336503	-0.332490	-0.328524	-0.324605	-0.320733
180.	180.	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
180.	240.	-0.313127	-0.305701	-0.298452	-0.291374	-0.284464	-0.277719	-0.271133	-0.264703	-0.258426	-0.252297	-0.246314
180.	300.	-0.294773	-0.286634	-0.278762	-0.271149	-0.263784	-0.256659	-0.249763	-0.243090	-0.236629	-0.230374	-0.224318
240.	60.	-0.365991	-0.363801	-0.361625	-0.359462	-0.357312	-0.355174	-0.353049	-0.350938	-0.348838	-0.346751	-0.344677
240.	120.	-0.337741	-0.333480	-0.329272	-0.325117	-0.321015	-0.316964	-0.312964	-0.309015	-0.305115	-0.301264	-0.297462
240.	180.	-0.316907	-0.311254	-0.305701	-0.300248	-0.294892	-0.289631	-0.284464	-0.279390	-0.274406	-0.269511	-0.264703
240.	240.	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
240.	300.	-0.274406	-0.266296	-0.258426	-0.250788	-0.243376	-0.236183	-0.229203	-0.222429	-0.215855	-0.209476	-0.203285
300.	60.	-0.329167	-0.327198	-0.325240	-0.323295	-0.321361	-0.319438	-0.317527	-0.315628	-0.313740	-0.311863	-0.309998
300.	120.	-0.224305	-0.221612	-0.218952	-0.216324	-0.213727	-0.211161	-0.208626	-0.206122	-0.203647	-0.201203	-0.198787
300.	180.	-0.295551	-0.290041	-0.284634	-0.279327	-0.274119	-0.269008	-0.263993	-0.259070	-0.254239	-0.249498	-0.244846
300.	240.	-0.277719	-0.271133	-0.264703	-0.258426	-0.252297	-0.246314	-0.240473	-0.234770	-0.229203	-0.223768	-0.218461
300.	300.	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Fig. 8-5

RULE 4B – MLE (Rule 1)

DIFFERENCE TABLE FOR P(CORRECT DECISION)

LAMBDA2 = 0.00100
M = 60.00000

LAMBDA1 VALUES

T2	T1	0.0012	0.0013	0.0014	0.0015	0.0016	0.0017	0.0018	0.0019	0.0020	0.0021	0.0022
60.	60.	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
60.	120.	0.000254	0.000295	0.000338	0.000383	0.000431	0.000480	0.000532	0.000586	0.000642	0.000699	0.000758
60.	180.	0.000569	0.000660	0.000756	0.000857	0.000963	0.001074	0.001189	0.001308	0.001431	0.001558	0.001688
60.	240.	0.000175	0.000217	0.000266	0.000320	0.000381	0.000447	0.000520	0.000598	0.000683	0.000774	0.000872
60.	300.	0.000182	0.000227	0.000279	0.000338	0.000403	0.000476	0.000555	0.000642	0.000736	0.000837	0.000946
120.	60.	-0.000214	-0.000231	-0.000247	-0.000263	-0.000279	-0.000295	-0.000310	-0.000326	-0.000341	-0.000356	-0.000370
120.	120.	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
120.	180.	0.000004	0.000005	0.000007	0.000008	0.000010	0.000011	0.000013	0.000015	0.000017	0.000020	0.000022
120.	240.	0.001655	0.001897	0.002148	0.002407	0.002674	0.002948	0.003227	0.003510	0.003798	0.004088	0.004381
120.	300.	0.002408	0.002742	0.003087	0.003439	0.003798	0.004162	0.004529	0.004898	0.005267	0.005637	0.006005
180.	60.	-0.000481	-0.000517	-0.000554	-0.000590	-0.000626	-0.000661	-0.000695	-0.000730	-0.000763	-0.000797	-0.000830
180.	120.	-0.000004	-0.000004	-0.000005	-0.000005	-0.000006	-0.000007	-0.000008	-0.000008	-0.000009	-0.000010	-0.000011
180.	180.	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
180.	240.	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000001	0.000001	0.000001
180.	300.	0.000037	0.000045	0.000055	0.000066	0.000077	0.000090	0.000104	0.000118	0.000134	0.000150	0.000168
240.	60.	-0.000125	-0.000135	-0.000144	-0.000154	-0.000163	-0.000172	-0.000181	-0.000190	-0.000199	-0.000207	-0.000216
240.	120.	-0.000000	-0.000001	-0.000001	-0.000001	-0.000001	-0.000001	-0.000001	-0.000001	-0.000001	-0.000001	-0.000001
240.	180.	-0.000000	-0.000000	-0.000000	-0.000000	-0.000000	-0.000000	-0.000000	-0.000000	-0.000000	-0.000000	-0.000000
240.	240.	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
240.	300.	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.000000
300.	60.	-0.000129	-0.000139	-0.000149	-0.000158	-0.000168	-0.000177	-0.000187	-0.000196	-0.000205	-0.000214	-0.000223
300.	120.	-0.002080	-0.002226	-0.002369	-0.002508	-0.002643	-0.002775	-0.002903	-0.003028	-0.003150	-0.003268	-0.003383
300.	180.	-0.000031	-0.000036	-0.000041	-0.000046	-0.000052	-0.000057	-0.000063	-0.000069	-0.000075	-0.000082	-0.000088
300.	240.	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
300.	300.	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Fig. 8.6

DIFFERENCE TABLE FOR P(CORRECT DECISION)

LAMBDA2 = 0.00100
M = 60.00000

LAMBDA1 VALUES

T2	T1	0.0012	0.0013	0.0014	0.0015	0.0016	0.0017	0.0018	0.0019	0.0020	0.0021	0.0022
60.	60.	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
60.	120.	-0.407477	-0.402573	-0.397724	-0.392930	-0.388191	-0.383505	-0.378873	-0.374293	-0.369766	-0.365290	-0.360865
60.	180.	-0.378306	-0.371366	-0.364538	-0.357819	-0.351208	-0.344703	-0.338304	-0.332008	-0.325815	-0.319722	-0.313727
60.	240.	-0.351117	-0.342446	-0.333957	-0.325648	-0.317513	-0.309551	-0.301757	-0.294128	-0.286661	-0.279352	-0.272198
60.	300.	-0.328188	-0.318398	-0.308883	-0.299636	-0.290648	-0.281912	-0.273418	-0.265161	-0.257131	-0.249323	-0.241730
120.	60.	0.412439	0.409954	0.407484	0.405029	0.402588	0.400163	0.397751	0.395355	0.392973	0.390605	0.388251
120.	120.	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
120.	180.	-0.357307	-0.350932	-0.344671	-0.338520	-0.332480	-0.326547	-0.320720	-0.314996	-0.309374	-0.303853	-0.298429
120.	240.	-0.330815	-0.322684	-0.314730	-0.306950	-0.299339	-0.291896	-0.284616	-0.277497	-0.270535	-0.263728	-0.257072
120.	300.	-0.304577	-0.294763	-0.285201	-0.275884	-0.266810	-0.257973	-0.249369	-0.240994	-0.232842	-0.224911	-0.217194
180.	60.	0.387689	0.385292	0.382910	0.380543	0.378191	0.375853	0.373530	0.371220	0.368926	0.366645	0.364379
180.	120.	0.361622	0.357307	0.353045	0.348833	0.344671	0.340559	0.336496	0.332481	0.328514	0.324595	0.320722
180.	180.	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
180.	240.	-0.313127	-0.305701	-0.298452	-0.291374	-0.284464	-0.277718	-0.271132	-0.264702	-0.258425	-0.252297	-0.246314
180.	300.	-0.2913[illegible]1	-0.282672	-0.274296	-0.266165	-0.258271	-0.250608	-0.243169	-0.235947	-0.228936	-0.222129	-0.215520
240.	60.	0.364348	0.362032	0.359731	0.357445	0.355173	0.352915	0.350672	0.348443	0.346228	0.344027	0.341840
240.	120.	0.340549	0.336484	0.332467	0.328498	0.324576	0.320701	0.316872	0.313088	0.309350	0.305655	0.302005
240.	180.	0.316907	0.311254	0.305701	0.300248	0.294892	0.289631	0.284464	0.279390	0.274406	0.269511	0.264703
240.	240.	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
240.	300.	-0.274406	-0.266296	-0.258426	-0.250788	-0.243376	-0.236183	-0.229203	-0.222429	-0.215855	-0.209476	-0.203285
300.	60.	0.344437	0.342356	0.340288	0.338233	0.336190	0.334160	0.332141	0.330135	0.328141	0.326159	0.324189
300.	120.	0.316574	0.312455	0.308389	0.304376	0.300415	0.296505	0.292645	0.288835	0.285075	0.281363	0.277699
300.	180.	0.298389	0.293[illegible]55	0.287816	0.28267[illegible]	0.277615	0.272649	0.267772	0.262981	0.258275	0.253653	0.249112
300.	240.	0.277719	0.271133	0.264703	0.258426	0.252297	0.246314	0.240473	0.234771	0.229203	0.223768	0.218461
300.	300.	0.	0.	0.0	0.	0.	0.0	0.0	0.0	0.0	0.0	0.0

Fig. 8-7

DECISION RULE MLE (Rule 1)
F(N2)=N2*T1/T2

TABLE OF VALUES FOR P(CORRECT DECISION) USING M.L.E.DECISION RULE

LAMBDA2 = 0.01000
M = 60.00000

LAMBDA1 VALUES

T2	T1	0.0010	0.0020	0.0030	0.0040	0.0050	0.0060	0.0070	0.0080	0.0090	0.0100	0.0110
60.	60.	0.699654	0.674508	0.650141	0.626537	0.603679	0.581551	0.560136	0.539417	0.519377	1.000000	0.518731
60.	120.	0.693426	0.662673	0.633204	0.604915	0.577724	0.551567	0.526393	0.502160	0.478833	1.000000	0.565217
60.	180.	0.680245	0.641568	0.607818	0.577677	0.550170	0.524588	0.500433	0.477362	0.455150	1.000000	0.587196
60.	240.	0.667024	0.620714	0.583566	0.553010	0.527001	0.503974	0.482786	0.462644	0.443035	1.000000	0.595624
60.	300.	0.654471	0.601714	0.562711	0.532316	0.508625	0.488844	0.471192	0.454413	0.437688	1.000000	0.597202
120.	60.	0.812909	0.777852	0.744184	0.711858	0.680830	0.651054	0.622487	0.595086	0.568810	1.000000	0.480528
120.	120.	0.809975	0.771303	0.733513	0.696706	0.660966	0.626357	0.592927	0.560712	0.529732	1.000000	0.528481
120.	180.	0.819301	0.784677	0.747314	0.708536	0.669318	0.630368	0.592187	0.555127	0.519423	1.000000	0.547378
120.	240.	0.812466	0.774262	0.735284	0.696004	0.656839	0.618151	0.580232	0.543324	0.507612	1.000000	0.559695
120.	300.	0.809068	0.773054	0.737024	0.699602	0.660808	0.621215	0.581529	0.542391	0.504308	1.000000	0.567358
180.	60.	0.874621	0.833779	0.794746	0.757451	0.721821	0.687788	0.655287	0.624253	0.594625	1.000000	0.460648
180.	120.	0.872004	0.827375	0.783677	0.741038	0.699748	0.659767	0.621230	0.584196	0.548704	1.000000	0.517581
180.	180.	0.874978	0.831589	0.787747	0.743934	0.700555	0.657950	0.616394	0.576109	0.537268	1.000000	0.535598
180.	240.	0.891376	0.857198	0.816101	0.770909	0.723492	0.675309	0.627461	0.580754	0.535753	1.000000	0.547773
180.	300.	0.884867	0.842856	0.796786	0.749410	0.702083	0.655480	0.609959	0.565751	0.523033	1.000000	0.557335
240.	60.	0.908011	0.863581	0.821253	0.780932	0.742528	0.705954	0.671126	0.637965	0.606395	1.000000	0.452262
240.	120.	0.921315	0.885165	0.847089	0.807834	0.768016	0.728137	0.688606	0.649743	0.611821	1.000000	0.460494
240.	180.	0.907311	0.860056	0.812999	0.766332	0.720284	0.675094	0.630990	0.588184	0.546860	1.000000	0.530770
240.	240.	0.915375	0.872380	0.826743	0.779434	0.731302	0.683078	0.635377	0.588707	0.543476	1.000000	0.541480
240.	300.	0.933846	0.900866	0.858781	0.810350	0.757965	0.703023	0.648930	0.595124	0.543121	1.000000	0.553149
300.	60.	0.925912	0.879143	0.834593	0.792422	0.752254	0.714079	0.677601	0.642328	0.610572	1.000000	0.450113
300.	120.	0.927[illegible]	0.87[illegible]	0.831321	0.784414	0.733593	0.694098	0.651110	0.609767	0.570162	1.000000	0.503615
300.	180.	0.930758	0.888147	0.852354	0.804064	0.754594	0.704971	0.655984	0.608228	0.562144	1.000000	0.523859
300.	240.	0.931361	0.876575	0.830519	0.791337	0.742553	0.693698	0.645235	0.597617	0.551239	1.000000	0.536491
300.	300.	0.941542	0.901528	0.856475	0.807752	0.756632	0.704261	0.651638	0.599613	0.548883	1.000000	0.546609

F(N2)=(N2+1)(T1+M)/(T2+M)-1

TABLE OF VALUES FOR P(CORRECT DECISION) USING DECISION RULE

LAMBDA2 = 0.1
M = 6.

LAMBDA1 VALUES

T2	T1	0.01	0.02	0.03	0.04	0.05	0.060	0.070	0.080	0.090	0.100	0.110
6.	6.	.699654	.6745 8	.65 141	.626537	.6 3679	0.581551	0.56 136	0.539417	0.519377	1.000000	0.518731
6.	12.	.9358 2	.878523	.824626	.774644	.728171	0.684857	0.644398	0.606529	0.571020	1.000000	0.493691
6.	18.	.95 7 5	.9 1934	.854 67	.8 737	.762 29	0.718174	0.675891	0.635240	0.596258	1.000000	0.476643
6.	24.	.98649	.953517	.90947	.8598 3	.8 7986	0.756165	0.705613	0.657045	0.610823	1.000000	0.474125
6.	3 .	.988374	.96 135	.919877	.872829	.82241	0.770945	0.719935	0.670293	0.622542	1.000000	0.466360
12.	6.	.954759	.9115 4	.87 152	.83 624	.792841	0.756731	0.722221	0.689244	0.657734	1.000000	0.491134
12.	12.	.8 9975	.7713 3	.733513	.69676	.66 966	0.626357	.592927	0.560712	0.529732	1.000000	0.528481
12.	18.	.945 9	.889745	.835 74	.781839	.73 543	.6815	.634883	0.590767	0.549158	1.000000	0.526746
12.	24.	.935 24	.88 331	.831384	.785533	.741334	0.698 95	.655574	0.613785	0.572874	1.000000	0.505522
12.	3 .	.9547 5	.9 8863	.862283	.815 58	.767463	0.719865	.672662	.626245	0.580969	1.000000	0.504988
18.	6.	.812381	.775244	.739748	.7 5825	.6734 7	0.642431	.612836	0.584563	0.557555	1.000000	0.492880
18.	12.	.9453 8	.892389	.841342	.792235	.745118	0.7 0 19	0.656947	0.615897	0.576851	1.000000	0.495361
18.	18.	.874978	.831589	.787747	.743934	.7 555	0.65795	.616394	.576109	0.537268	1.000000	0.535598
18.	24.	.95689	.9 834	.856331	.8 2555	.748386	.69489	.642863	0.592868	0.545279	1.000000	0.541893
18.	3 .	.951 55	.9 2911	.854885	.8 6565	.757868	0.7 8974	.66 235	.612 85	.564968	1.000000	0.524583
24.	6.	.95 745	.9 3872	.859269	.816829	.776449	0.738 32	.7 1483	0.666716	0.633643	1.000000	0.427737
24.	12.	.933739	.871521	.813114	.758362	.7 6881	.658659	.613454	.571 98	0.531427	1.000000	0.540460
24.	18.	.945195	.8917 2	.839432	.788411	.738726	0.69 498	.643858	0.598935	0.555843	1.000000	0.524492
24.	24.	.915375	.87238	.826743	.779434	.7313 2	0.683 78	.635377	.588707	0.543476	1.000000	0.541480
24.	3 .	.967449	.92576	.877223	.824 12	.768 86	0.711121	.654484	.599239	0.546169	1.000000	0.551478
3 .	6.	.843871	.8 1914	.762 5	.724 48	.687948	0.653617	.62 97	0.589927	.56041	1.000000	0.494335
3 .	12.	.871 16	.818715	.7685 9	.72 46	.6746 1	0.63 942	.589471	.55 161	0.512974	1.000000	0.555248
3 .	18.	.943719	.8873	.831 11	.776926	.72 733	.672357	.623 39	0.575958	0.531234	1.000000	0.550899
3 .	24.	.951543	.9 1976	.851636	.8 869	.75 57	.699596	.649874	.6 1266	0.554111	1.000000	0.534713
3 .	3 .	.961542	.9 1128	.856475	.8 7752	.756632	.7 4261	.651639	.599613	.548883	1.000000	0.546609

Fig. 8-9

DECISION RULE 4B
F(N2)=N2(T1+M)/(T2+M)

TABLE OF VALUES FOR P(CORRECT DECISION) USING DECISION RULE

LAMBDA2 = 0.01000
M = 40.00000

LAMBDA1 VALUES

T2	T1	0.0010	0.0020	0.0030	0.0040	0.0050	0.0060	0.0070	0.0080	0.0090	0.0100	0.0110
60.	60.	0.699654	0.674508	0.650141	0.626537	0.603679	0.581551	0.560136	0.539417	0.519377	1.000000	0.518731
60.	120.	0.692362	0.658849	0.625468	0.592545	0.560330	0.529013	0.498736	0.469597	0.441661	1.000000	0.610460
60.	180.	0.677842	0.633204	0.591186	0.551567	0.514161	0.478833	0.445478	0.414012	0.384362	1.000000	0.669760
60.	240.	0.666409	0.616725	0.572649	0.532012	0.493697	0.457198	0.422341	0.389120	0.357593	1.000000	0.700118
60.	300.	0.653837	0.597425	0.550170	0.508350	0.469871	0.433659	0.399236	0.366461	0.335344	1.000000	0.721643
120.	60.	0.819172	0.789754	0.761255	0.733619	0.706831	0.680874	0.655733	0.631391	0.607829	1.000000	0.437020
120.	120.	0.809975	0.771303	0.733513	0.696706	0.660965	0.626357	0.592927	0.560712	0.529732	1.000000	0.528481
120.	180.	0.819211	0.784068	0.745566	0.705015	0.663476	0.621791	0.580618	0.540459	0.501683	1.000000	0.570753
120.	240.	0.808360	0.761225	0.711855	0.662517	0.614497	0.568481	0.524800	0.483581	0.444836	1.000000	0.625452
120.	300.	0.803015	0.754840	0.705828	0.656839	0.608588	0.561639	0.516419	0.473238	0.432306	1.000000	0.642349
180.	60.	0.886839	0.857002	0.827854	0.799403	0.771656	0.744617	0.718287	0.692666	0.667753	1.000000	0.379971
180.	120.	0.872526	0.829261	0.787507	0.747232	0.708410	0.671020	0.635045	0.600469	0.567276	1.000000	0.495032
180.	180.	0.874978	0.831589	0.787747	0.743934	0.700555	0.657950	0.616394	0.576109	0.537263	1.000000	0.535598
180.	240.	0.891767	0.857093	0.815635	0.769727	0.721176	0.671456	0.621738	0.572929	0.525711	1.000000	0.562152
180.	300.	0.884419	0.840181	0.790000	0.737248	0.684013	0.631582	0.580745	0.531992	0.485621	1.000000	0.599329
240.	60.	0.923473	0.892909	0.862970	0.833674	0.805050	0.777096	0.749826	0.723247	0.697361	1.000000	0.352323
240.	120.	0.921723	0.886641	0.850091	0.812660	0.774823	0.737012	0.699526	0.662641	0.626572	1.000000	0.442492
240.	180.	0.907363	0.860411	0.814020	0.768395	0.723717	0.680145	0.637619	0.596859	0.557369	1.000000	0.516892
240.	240.	0.915375	0.872330	0.826743	0.779434	0.731302	0.683078	0.635377	0.588707	0.543476	1.000000	0.541480
240.	300.	0.933845	0.900843	0.853547	0.809921	0.756974	0.701750	0.645877	0.590625	0.536994	1.000000	0.562685
300.	60.	0.944634	0.914695	0.885155	0.856129	0.827640	0.799715	0.772377	0.745644	0.719530	1.000000	0.330794
300.	120.	0.939136	0.903287	0.865791	0.827812	0.789706	0.751774	0.714269	0.677402	0.641348	1.000000	0.427798
300.	180.	0.941326	0.903492	0.862697	0.819869	0.775825	0.731271	0.686786	0.642864	0.599893	1.000000	0.482004
300.	240.	0.931567	0.886647	0.839813	0.792036	0.744025	0.696145	0.648877	0.602605	0.557652	1.000000	0.527286
300.	300.	0.941542	0.901523	0.856475	0.807762	0.756632	0.704261	0.651638	0.599613	0.548883	1.000000	0.546609

Fig. 8-18

DECISION RULE
F(N2)=((N2+.11)*(M+T1+111)/(M+T2+111))-.11

TABLE OF VALUES FOR P(CORRECT DECISION) USING DECISION RULE

LAMBDA2 = 0.01000
M = 60.00000

LAMBDA1 VALUES

T2	T1	0.0010	0.0020	0.0030	0.0040	0.0050	0.0060	0.0070	0.0080	0.0090	0.0100	0.0110
60.	60.	0.699654	0.674508	0.650141	0.626537	0.603673	0.581551	0.560136	0.539417	0.519377	1.000000	0.519731
60.	120.	0.935725	0.874612	0.816637	0.761752	0.709892	0.660978	0.614916	0.571608	0.530945	1.000000	0.542885
60.	180.	0.904855	0.817182	0.737000	0.664077	0.598024	0.538371	0.484607	0.436214	0.392690	1.000000	0.681617
60.	240.	0.374733	0.762908	0.664092	0.577505	0.502030	0.436457	0.379596	0.330333	0.287660	1.000000	0.781388
60.	300.	0.856571	0.744732	0.653388	0.576034	0.508848	0.449557	0.396769	0.349585	0.307375	1.000000	0.763982
120.	60.	0.677379	0.656470	0.636073	0.616185	0.596798	0.577908	0.559509	0.541593	0.524153	1.000000	0.509327
120.	120.	0.839975	0.771303	0.733513	0.696706	0.660956	0.626357	0.592927	0.560712	0.529732	1.000000	0.528481
120.	180.	0.944999	0.889114	0.833231	0.778062	0.724170	0.671990	0.621349	0.573980	0.528544	1.000000	0.554706
120.	240.	0.926431	0.851933	0.779555	0.707812	0.640741	0.577984	0.519872	0.466503	0.417806	1.000000	0.666371
120.	300.	0.903527	0.813283	0.735907	0.659595	0.590325	0.527645	0.470926	0.419568	0.373048	1.000000	0.707169
180.	60.	0.816630	0.798189	0.779464	0.760540	0.741490	0.722381	0.703271	0.684214	0.665256	1.000000	0.372195
180.	120.	0.799232	0.764314	0.730053	0.696539	0.663842	0.632027	0.601139	0.571217	0.542287	1.000000	0.512579
180.	180.	0.874378	0.831599	0.787747	0.743934	0.700555	0.657950	0.616394	0.576109	0.537268	1.000000	0.535598
180.	240.	0.956681	0.908220	0.855834	0.801269	0.745811	0.690520	0.636239	0.583629	0.533195	1.000000	0.559885
180.	300.	0.945201	0.882893	0.816977	0.750476	0.685502	0.623396	0.564917	0.510416	0.459980	1.000000	0.629084
240.	60.	0.895783	0.881486	0.866494	0.850905	0.834805	0.818279	0.801399	0.784239	0.766859	1.000000	0.268323
240.	120.	0.882936	0.855752	0.827070	0.799869	0.771254	0.742349	0.713281	0.684184	0.655184	1.000000	0.402046
240.	180.	0.869477	0.828781	0.787555	0.746500	0.705658	0.665420	0.626030	0.587685	0.550545	1.000000	0.519650
240.	240.	0.915375	0.872283	0.826743	0.779434	0.731302	0.683078	0.635377	0.588707	0.543476	1.000000	0.541480
240.	300.	0.967449	0.925735	0.877078	0.823540	0.766970	0.708971	0.650387	0.593816	0.538613	1.000000	0.563754
300.	60.	0.927784	0.905242	0.882935	0.861607	0.839400	0.816351	0.794492	0.772855	0.751455	1.000000	0.290477
300.	120.	[illegible]	[illegible]	0.861052	0.852142	[illegible]	0.784474	0.762550	0.731342	0.700112	1.000000	0.361594
300.	180.	[illegible]	[illegible]	0.857765	0.822153	[illegible]	0.745570	0.705866	0.665822	0.625893	1.000000	0.452119
300.	240.	[illegible]	[illegible]	[illegible]	0.732621	0.706715	[illegible]	[illegible]	0.599973	0.556271	1.000000	0.526416
300.	300.	[illegible]	[illegible]	[illegible]	0.807752	[illegible]	0.704261	[illegible]	0.595613	0.548883	1.000000	0.546609

Fig. 8-4

RUL 4A- ML (Rule 1)

DIFFERENCE TABLE FOR P(CORRECT DECISION)

LAMBDA2 = 0.1300
M = 6.0000

LAMBDA1 VALUES

T2	T1	0.0010	0.0020	0.0030	0.0040	0.0050	0.0060	0.0070	0.0080	0.0090	0.0100	0.0110
60.	60.	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
60.	120.	0.243376	0.21585	0.191422	.169729	0.15.447	0.133290	0.118 5	0.104369	0.092187	0.0	-0.071526
60.	180.	0.27 459	0.26 365	.246249	.229693	.21186	0.193585	0.175458	0.157878	0.141108	0.0	-0.110553
60.	240.	0.319466	0.3328 4	0.3259 4	.3 6793	0.28 985	0.252191	0.222827	0.194401	0.167789	0.0	-0.121499
60.	300.	0.3344 4	0.358421	0.357566	.34 514	0.313785	0.282101	0.248743	0.215881	0.184854	0.0	-0.130842
120.	60.	0.141849	.133652	0.125969	.118766	0.112 11	0.105677	0.099734	0.094158	0.088924	0.0	-0.079395
120.	120.	0.0	0.0	0.0	.0	.	0.0	0.0	0.0	0.0	0.0	0.0
120.	180.	0.125789	0.1 5 68	0. 8776	0. 7332 2	0. 61224	0.051132	0.042696	0.035641	0.029735	0.0	-0.020632
120.	240.	0.122558	0.1 6. 69	0. 961	0. 89529	0. 84495	0.079944	0. 75341	0.070461	0.065261	0.0	-0.054173
120.	300.	0.145637	0.1358 9	0.125259	.115457	0.1 6656	0.098649	0. 91133	0.083854	0.076661	0.0	-0.062370
180.	60.	-0. 6224 -	. 58534-	0. 54998-	. 51625-	. 48414-	0.045357-	0.042451-	0.039690-	0.037071	0.0	0.032232
180.	120.	0. 733 4	0. 65 15	0. 57665	0. 51148	. 45371	0. 40252	0. 35717	0.031701	0.028147	0.0	-0.022220
180.	180.	0.	.	0.	.	.	0.0	0.0	0.0	0.0	0.0	0.0
180.	240.	0. 65 14	0. 51142	0. 4 23	. 31646	0. 24893	0.019582	0. 154 3	0.012114	0.009526	0.0	-0.005880
180.	300.	. 66188	0. 6 55	. 53 99	. 57155	. 55784	0. 53494	0. 5 276	0.046333	0.041935	0.0	-0.032752
240.	60.	0. 42734	0. 4 29	. 38 16	. 35897	0. 33921	0. 32 78	0. 3 357	0.028750	0.027248	0.0	-0.024526
240.	120.	0. 12423-	. 13644-	0. 33974-	. 49532-	. 61135-	0. 69479-	. 75152-	.078651-	0.08 394	0.	0.079966
240.	180.	. 37887	. 31646	. 26433	. 22 79	0. 18442	0. 154 5	. 12868	0.010751	0.008983	0.	-0.006278
240.	240.	.	.	.	.	.	0.	0.	0.0	0.0	0.	0.0
240.	300.	0. 336 3	0. 24894	0. 18442	. 13662	. 1 121	0. 7498	0. 5554	0.004115	0.003048	0.	-0.001671
300.	60.	- . 82 41-	. 77229-	. 72678-	0. 68374-	. 643.6-	. 64462-	. 56831-	. 53401-	0.050162	0.	0.044222
300.	120.	- . 56 77-	. 6 223-	0. 62811-	. 63954-	. 63992-	. 63156-	. 6164 -	. 59605-	0.057188	0.	0.051634
300.	180.	. 3951-	. 1 768-	. 2 743-	0. 27139-	. 3 861-	. 32614-	. 32944-	.03227 -	0.03091	0.	0.027140
300.	240.	. 19582	0. 154 4	. 12117	. 9532	. 7498	0. 5898	. 464	. 3650	0.002871	0.	-0.001778
300.	300.	.	.	.	.	.	0.	0.	.0	0.0	0.	0.0

DIFFERENCE TABLE FOR P(CORRECT DECISION)

LAMBDA2 = 0.01000
M = 60.00000

LAMBDA1 VALUES

T2	T1	0.0010	0.0020	0.0030	0.0040	0.0050	0.0060	0.0070	0.0080	0.0090	0.0100	0.0110
60.	60.	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
60.	120.	-0.001064	-0.003825	-0.007736	-0.012370	-0.017394	-0.022554	-0.027657	-0.032563	-0.037171	0.0	0.045244
60.	180.	-0.002363	-0.008364	-0.016632	-0.026110	-0.036009	-0.045755	-0.054955	-0.063350	-0.070788	0.0	0.082564
60.	240.	-0.000615	-0.003989	-0.010917	-0.020998	-0.033303	-0.046776	-0.060445	-0.073524	-0.085442	0.0	0.104494
60.	300.	-0.000634	-0.004289	-0.012141	-0.023966	-0.038753	-0.055186	-0.071956	-0.087952	-0.102344	0.0	0.124442
120.	60.	0.006223	0.011902	0.017071	0.021760	0.026001	0.029820	0.033246	0.036304	0.039019	0.0	-0.043508
120.	120.	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
120.	180.	-0.000090	-0.000609	-0.001748	-0.003521	-0.005842	-0.008576	-0.011568	-0.014668	-0.017740	0.0	0.023375
120.	240.	-0.004106	-0.013037	-0.023430	-0.033487	-0.042343	-0.049670	-0.055432	-0.059743	-0.062776	0.0	0.065757
120.	300.	-0.006053	-0.018214	-0.031195	-0.042762	-0.052219	-0.059576	-0.065110	-0.069153	-0.072002	0.0	0.074991
180.	60.	0.012217	0.023224	0.033108	0.041953	0.049836	0.056829	0.063000	0.068413	0.073127	0.0	-0.080677
180.	120.	0.000522	0.001886	0.003830	0.006144	0.008663	0.011253	0.013815	0.016273	0.018572	0.0	-0.022548
180.	180.	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
180.	240.	-0.000009	-0.000115	-0.000466	-0.001183	-0.002316	-0.003852	-0.005722	-0.007824	-0.010042	0.0	0.014379
180.	300.	-0.000448	-0.002675	-0.006786	-0.012162	-0.019071	-0.023898	-0.029214	-0.033759	-0.037412	0.0	0.041993
240.	60.	0.015467	0.029326	0.041717	0.052746	0.062522	0.071142	0.078700	0.085282	0.090966	0.0	-0.099939
240.	120.	0.000408	0.001476	0.003003	0.004826	0.006817	0.008875	0.010920	0.012893	0.014750	0.0	-0.018002
240.	180.	0.000052	0.000355	0.001021	0.002063	0.003432	0.005051	0.006828	0.008675	0.010509	0.0	-0.013878
240.	240.	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
240.	300.	-0.000001	-0.000023	-0.000135	-0.000429	-0.000992	-0.001867	-0.003052	-0.004499	-0.006128	0.0	0.009536
300.	60.	0.018771	0.035542	0.050471	0.063707	0.075386	0.085636	0.094576	0.102316	0.108958	0.0	-0.119319
300.	120.	0.012794	0.024279	0.034470	0.043398	0.051113	0.057676	0.063158	0.067635	0.071186	0.0	-0.075826
300.	180.	0.001553	0.005350	0.010343	0.015803	0.021272	0.026300	0.030805	0.034635	0.037748	0.0	-0.041855
300.	240.	0.000006	0.000072	0.000295	0.000749	0.001460	0.002448	0.003642	0.004988	0.006412	0.0	-0.009205
300.	300.	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Fig. 8-13

DIFFERENCE TABLE FOR P(CORRECT DECISION)

LAMBDA2 = 0.01000
N = 60.00000

LAMBDA1 VALUES

T2	T1	0.0010	0.0020	0.0030	0.0040	0.0050	0.0060	0.0070	0.0080	0.0090	0.0100	0.0110
60.	60.	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
60.	120.	0.242299	0.211939	0.183433	0.156837	0.132169	0.109411	0.088523	0.069448	0.052113	0.0	-0.022331
60.	180.	0.224610	0.175614	0.129182	0.086399	0.047855	0.013733	-0.015827	-0.041149	-0.062460	0.0	0.094421
60.	240.	0.207769	0.142195	0.080526	0.024495	-0.024971	-0.067517	-0.103190	-0.132311	-0.155374	0.0	0.185764
60.	300.	0.202101	0.143018	0.091077	0.043718	0.000223	-0.039287	-0.074423	-0.104828	-0.130313	0.0	0.166780
120.	60.	-0.135530	-0.121382	-0.108111	-0.095674	-0.084032	-0.073146	-0.062978	-0.053494	-0.044657	0.0	0.028799
120.	120.	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
120.	180.	0.125698	0.104437	0.085917	0.069526	0.054852	0.041623	0.029662	0.018853	0.009121	0.0	0.007328
120.	240.	0.113965	0.077671	0.043271	0.011803	-0.016099	-0.040166	-0.060360	-0.076821	-0.089806	0.0	0.106676
120.	300.	0.099459	0.046229	-0.001117	-0.040017	-0.070433	-0.093570	-0.110603	-0.122822	-0.131260	0.0	0.139811
180.	60.	-0.057991	-0.035591	-0.015283	0.003089	0.019669	0.034592	0.047984	0.059960	0.070631	0.0	-0.088453
180.	120.	-0.072772	-0.063060	-0.053624	-0.044550	-0.035905	-0.027740	-0.020091	-0.012979	-0.006417	0.0	-0.005052
180.	180.	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
180.	240.	0.065005	0.051022	0.039733	0.030358	0.022319	0.015211	0.008779	0.002875	-0.002568	0.0	0.012111
180.	300.	0.060334	0.040037	0.020191	0.001067	-0.016581	-0.032094	-0.045042	-0.055335	-0.063053	0.0	0.071749
240.	60.	-0.012228	0.017905	0.045241	0.069972	0.092277	0.112324	0.130273	0.146273	0.160464	0.0	-0.183939
240.	120.	-0.038479	-0.029412	-0.019019	-0.007965	0.003238	0.014211	0.024675	0.034436	0.043362	0.0	-0.058448
240.	180.	-0.037834	-0.031276	-0.025344	-0.019833	-0.014627	-0.009673	-0.004960	-0.000500	0.003685	0.0	-0.011120
240.	240.	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
240.	300.	0.033602	0.024869	0.018297	0.013190	0.009005	0.005347	0.001958	-0.001308	-0.004504	0.0	0.010605
300.	60.	0.001872	0.026199	0.048251	0.068185	0.086146	0.102271	0.116691	0.129527	0.140892	0.0	-0.159637
300.	120.	0.003306	0.027381	0.049723	0.068726	0.085204	0.099376	0.111439	0.121575	0.129950	0.0	-0.142031
300.	180.	-0.017718	-0.006940	0.005411	0.017974	0.029881	0.040608	0.049882	0.057593	0.063748	0.0	-0.071739
300.	240.	-0.019576	-0.015328	-0.011891	-0.008708	-0.005844	-0.003079	-0.000348	0.002362	0.005032	0.0	-0.010075
300.	300.	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Fig. 8-14

TABLE OF VALUES FOR P(CORRECT DECISION), DECISION RULE 2.

LAMBDA2 = 0.001000

M = 60.000000

LAMBDA1 VALUES

N	0.001200	0.001300	0.001400	0.001500	0.001600	0.001700	0.001800	0.001900	0.002000	0.002100	0.002200
1.00	0.505617	0.508400	0.511167	0.513917	0.516650	0.519367	0.522068	0.524753	0.527422	0.530075	0.532712
2.00	0.510562	0.515754	0.520888	0.525964	0.530984	0.535947	0.540855	0.545708	0.550507	0.555252	0.559944
3.00	0.514955	0.522256	0.529442	0.536514	0.543476	0.550328	0.557072	0.563710	0.570243	0.576674	0.583005
4.00	0.518893	0.528060	0.537047	0.545856	0.554491	0.562957	0.571255	0.579391	0.587366	0.595185	0.602850
5.00	0.522453	0.533289	0.543875	0.554214	0.564314	0.574180	0.583817	0.593231	0.602427	0.611411	0.620187
6.00	0.525697	0.538042	0.550062	0.561768	0.573165	0.584264	0.595072	0.605596	0.615844	0.625823	0.635540

Fig. 8-15

TABLE OF VALUES FOR P(CORRECT DECISION), DECISION RULE 2

LAMBDA2 = 0.010000

M = 60.000000

LAMBDA1 VALUES

N	0.001000	0.002000	0.003000	0.004000	0.005000	0.006000	0.007000	0.008000	0.009000	0.010000	0.011000
1.00	0.696476	0.669054	0.643229	0.618908	0.596003	0.574432	0.554118	0.534986	0.516969	0.500000	0.515980
2.00	0.803188	0.759967	0.719532	0.681690	0.646262	0.613085	0.582007	0.552885	0.525590	0.500001	0.523996
3.00	0.866482	0.817254	0.769941	0.724721	0.681702	0.640933	0.602418	0.566129	0.532014	0.500001	0.529993
4.00	0.906656	0.857093	0.807193	0.757878	0.709813	0.663469	0.619161	0.577087	0.537354	0.500001	0.534984
5.00	0.933415	0.886427	0.836346	0.784882	0.733337	0.682680	0.633611	0.586618	0.542019	0.500001	0.539346
6.00	0.951841	0.908772	0.859905	0.807562	0.753618	0.699542	0.646448	0.595149	0.546211	0.500002	0.543268

Fig. 8-16

PLAY THE LOSER

TABLE OF VALUES FOR LAMBDA2 = 0.001000

M = 60.000000

LAMBDA1 VALUES

	0.001100	0.001200	0.001300	0.001400	0.001500	0.001600	0.001700	0.001800	0.001900	0.002000	0.002100
U1 =	0.523157	0.544167	0.563316	0.58[illegible]841	0.596943	0.611789	0.625522	0.638263	0.650118	0.661175	0.671515
P(1) =	0.939831	0.939437	0.939044	0.938622	0.938171	0.937693	0.937186	0.936652	0.936090	0.935503	0.934888
P(2) =	0.934511	0.928921	0.923364	0.917841	0.912356	0.906892	0.901467	0.896075	0.890714	0.885386	0.880090
P(1,2) =	0.937279	0.934643	0.932197	0.929911	0.927764	0.925736	0.923810	0.921974	0.920214	0.918522	0.916887
RATIO =	1.041904	1.079151	1.112325	1.141920	1.168325	1.191906	1.212942	1.231709	1.248417	1.263279	1.276450

Fig. 8-17

PLAY THE LOSER

TABLE OF VALUES FOR LAMBDA2 = 0.010000
M = 60.000000

LAMBDA1 VALUES

	0.001000	0.002000	0.003000	0.004000	0.005000	0.006000
U1 =	0.101958	0.181366	0.245439	0.298604	0.343730	0.382756
R(1) =	0.547862	0.545163	0.540919	0.535321	0.528540	0.520734
R(2) =	0.826962	0.778803	0.733449	0.690737	0.650511	0.612628
R(1,2) =	0.798505	0.736429	0.686195	0.644329	0.608586	0.577455
RATIO =	1.164452	1.191569	1.177280	1.150059	1.119961	1.090799

	0.007000	0.008000	0.009000	0.010000	0.011000
U1 =	0.417038	0.447555	0.475027	0.500000	0.522890
R(1) =	0.512045	0.502601	0.492521	0.481911	0.470866
R(2) =	0.576952	0.543353	0.511711	0.481911	0.453847
R(1,2) =	0.549883	0.525114	0.502595	0.481911	0.462746
RATIO =	1.063984	1.039940	1.018666	1.000000	1.015346

Fig. 8-18

DECISION RULE (ML1) 1
F(N2)=N2*T1/T2

TABLE OF VALUES FOR THE RATIO OF M.L.E. UNRELIABILITY TO MINIMUM SYSTEM UNRELIABILITY

LAMBDA2 = 0.01000
M = 60.00000

LAMBDA1 VALUES

T2	T1	0.0012	0.0013	0.0014	0.0015	0.0016	0.0017	0.0018	0.0019	0.0020	0.0021	0.0022
60.	60.	1.085843	1.126448	1.165502	1.202984	1.238912	1.273273	1.306093	1.337367	1.367134	1.395393	1.422179
60.	120.	1.080863	1.118525	1.154366	1.188397	1.220654	1.251151	1.279939	1.307032	1.332489	1.356334	1.378619
60.	180.	1.075986	1.110818	1.143613	1.174406	1.203263	1.230225	1.255363	1.278724	1.300383	1.320386	1.338803
60.	240.	1.071414	1.103638	1.133647	1.161512	1.187325	1.211155	1.233098	1.253222	1.271622	1.288367	1.303546
60.	300.	1.067157	1.096987	1.124472	1.149710	1.172821	1.193901	1.213068	1.230412	1.246046	1.260060	1.272552
120.	60.	1.089965	1.132479	1.173344	1.212537	1.250081	1.285962	1.320210	1.352820	1.383835	1.413255	1.441117
120.	120.	1.084979	1.124543	1.162188	1.197915	1.231770	1.263764	1.293951	1.322351	1.349020	1.373987	1.397305
120.	180.	1.081305	1.118713	1.154017	1.187242	1.218448	1.247670	1.274975	1.300404	1.324027	1.345890	1.366063
120.	240.	1.077023	1.111986	1.144681	1.175163	1.203518	1.229805	1.254113	1.276504	1.297069	1.315871	1.332994
120.	300.	1.073249	1.106116	1.136615	1.164828	1.190866	1.214808	1.236766	1.256816	1.275065	1.291591	1.306490
180.	60.	1.094103	1.138567	1.181304	1.222290	1.261548	1.299066	1.334871	1.368963	1.401383	1.432134	1.461255
180.	120.	1.087873	1.128657	1.167379	1.204050	1.238720	1.271408	1.302174	1.331042	1.358081	1.383320	1.406822
180.	180.	1.084209	1.122853	1.159259	1.193459	1.225520	1.255483	1.283424	1.309388	1.333458	1.355680	1.376131
180.	240.	1.082[illegible]73	1.119460	1.154498	1.187233	1.217743	1.246081	1.272332	1.296553	1.318833	1.339231	1.357835
180.	300.	1.077862	1.112825	1.145269	1.175262	1.202911	1.228292	1.251513	1.272654	1.291822	1.309099	1.324584
240.	60.	1.098028	1.144347	1.188866	1.231561	1.272453	1.311534	1.348831	1.384342	1.418112	1.450144	1.480476
240.	120.	1.091749	1.134358	1.174829	1.213173	1.249438	1.283647	1.315858	1.346099	1.374435	1.400901	1.425561
240.	180.	1.085631	1.124678	1.161304	1.195552	1.227510	1.257229	1.284800	1.310279	1.333761	1.355306	1.375001
240.	240.	1.083517	1.121342	1.156649	1.189499	1.219989	1.248184	1.274183	1.298056	1.319906	1.339802	1.357840
240.	300.	1.082998	1.120485	1.155391	1.187793	1.217781	1.245424	1.270828	1.294062	1.315230	1.334408	1.351696
300.	60.	1.101729	1.149796	1.195994	1.240300	1.282738	1.323293	1.361998	1.398849	1.433893	1.467134	1.498610
300.	120.	1.09474[illegible]	1.138679	1.180376	1.219845	1.257139	1.292282	1.325340	1.356340	1.385357	1.412424	1.437612
300.	180.	1.089203	1.129919	1.168137	1.203898	1.237292	1.268370	1.297226	1.323913	1.348531	1.371140	1.391829
300.	240.	1.083371	1.120746	1.155388	1.187383	1.216852	1.243881	1.268591	1.291071	1.311443	1.329794	1.346233
300.	300.	1.082891	1.119976	1.154299	1.185948	1.215045	1.241681	1.265979	1.288029	1.307956	1.325849	1.341822

Fig. 8-1 (A)

DECISION RULE 4A
F(N2)=(N2+1)(T1+M)/(T2+M)-1

TABLE OF VALUES FOR THE RATIO OF UNRELIABILITY TO MINIMUM SYSTEM UNRELIABILITY

LAMBDA2 = 0.00100
M = 60.00000

LAMBDA1 VALUES

T2	T1	0.0012	0.0013	0.0014	0.0015	0.0016	0.0017	0.0018	0.0019	0.0020	0.0021	0.0022
60.	60.	1.085843	1.126448	1.165532	1.202984	1.238912	1.273273	1.306093	1.337367	1.367134	1.395393	1.422179
60.	120.	1.151662	1.222152	1.289145	1.352654	1.412763	1.469500	1.522965	1.573197	1.620308	1.664350	1.705423
60.	180.	1.156096	1.229099	1.298759	1.365057	1.428048	1.487735	1.544191	1.597434	1.647553	1.694582	1.738606
60.	240.	1.168029	1.247620	1.324145	1.397492	1.467637	1.534502	1.598099	1.658376	1.715379	1.769090	1.819552
60.	300.	1.168971	1.249194	1.326458	1.400643	1.471722	1.539604	1.604295	1.665735	1.723963	1.778944	1.830718
120.	60.	1.161618	1.237988	1.311393	1.381796	1.449231	1.513676	1.575187	1.633755	1.689451	1.742285	1.792318
120.	120.	1.084979	1.124543	1.162188	1.197915	1.231770	1.263764	1.293951	1.322351	1.349020	1.373987	1.397305
120.	180.	1.143349	1.208983	1.270719	1.328618	1.382808	1.433362	1.480419	1.524061	1.564435	1.601628	1.635776
120.	240.	1.135043	1.195970	1.252707	1.305376	1.354154	1.399165	1.440590	1.478554	1.513239	1.544770	1.573310
120.	300.	1.146362	1.213558	1.276851	1.336271	1.391924	1.443863	1.492208	1.537029	1.578460	1.616581	1.651521
180.	60.	1.149436	1.220046	1.287913	1.353001	1.415341	1.474916	1.531776	1.585913	1.637396	1.686227	1.732470
180.	120.	1.150665	1.220567	1.286916	1.349732	1.409104	1.465065	1.517718	1.567109	1.613352	1.656505	1.696671
180.	180.	1.084209	1.122853	1.159259	1.193459	1.225520	1.255483	1.283424	1.309388	1.333458	1.355680	1.376131
180.	240.	1.136444	1.198094	1.255548	1.308917	1.358363	1.404001	1.446005	1.484491	1.519638	1.551564	1.580431
180.	300.	1.129046	1.186557	1.239654	1.288501	1.333306	1.374237	1.411498	1.445247	1.475692	1.502979	1.527302
240.	60.	1.161579	1.237926	1.311305	1.381680	1.449083	1.513498	1.574976	1.633508	1.689171	1.741967	1.791964
240.	120.	1.150395	1.220139	1.286315	1.348948	1.408126	1.463883	1.516326	1.565499	1.611519	1.654444	1.694380
240.	180.	1.140658	1.204741	1.264829	1.320942	1.373283	1.421924	1.467012	1.508646	1.546983	1.582125	1.614215
240.	240.	1.083517	1.121342	1.156649	1.189499	1.219989	1.248184	1.274183	1.298056	1.319906	1.339802	1.357840
240.	300.	1.130645	1.188977	1.242889	1.292526	1.338090	1.379727	1.417644	1.451986	1.482957	1.510702	1.535405
300.	60.	1.158885	1.233959	1.306115	1.375315	1.441596	1.504935	1.565389	1.622946	1.677679	1.729595	1.778757
300.	120.	1.133688	1.195683	1.254510	1.310185	1.362790	1.412356	1.458974	1.502687	1.543597	1.581756	1.617258
300.	180.	1.140522	1.204525	1.264508	1.320552	1.372798	1.421338	1.466324	1.507852	1.546082	1.581116	1.613096
300.	240.	1.131594	1.190488	1.245012	1.295307	1.341570	1.383944	1.422626	1.457759	1.489541	1.518115	1.543658
300.	300.	1.082891	1.119976	1.154299	1.185948	1.215045	1.241681	1.265979	1.288029	1.307956	1.325849	1.341822

F(N2)=N2(T1+M)/(T2+M)

TABLE OF VALUES FOR THE RATIO OF UNRELIABILITY TO MINIMUM SYSTEM UNRELIABILITY

LAMBDA2 = 0.00100
M = 60.00000

LAMBDA1 VALUES

T2	T1	0.0012	0.0013	0.0014	0.0015	0.0016	0.0017	0.0018	0.0019	0.0020	0.0021	0.0022
60.	60.	1.085843	1.126448	1.165502	1.202984	1.238912	1.273273	1.306093	1.337367	1.367134	1.395393	1.422179
60.	120.	1.080818	1.118448	1.154253	1.188237	1.220441	1.250879	1.279597	1.306615	1.331990	1.355746	1.377934
60.	180.	1.075888	1.110648	1.143357	1.174048	1.202787	1.229613	1.254601	1.277795	1.299271	1.319075	1.337278
60.	240.	1.071385	1.103580	1.133558	1.161378	1.187139	1.210901	1.232766	1.252797	1.271091	1.287716	1.302757
60.	300.	1.067126	1.096928	1.124377	1.149568	1.172622	1.193630	1.212712	1.229956	1.245476	1.259356	1.271697
120.	60.	1.090003	1.132539	1.173427	1.212647	1.250218	1.286129	1.320408	1.353051	1.384099	1.413554	1.441452
120.	120.	1.084979	1.124543	1.162188	1.197915	1.231770	1.263764	1.293951	1.322351	1.349020	1.373987	1.397305
120.	180.	1.081305	1.118711	1.154015	1.187239	1.218444	1.247664	1.274966	1.300392	1.324014	1.345875	1.366043
120.	240.	1.076735	1.111498	1.143954	1.174157	1.202196	1.228129	1.252047	1.274012	1.294118	1.312431	1.329036
120.	300.	1.072831	1.105411	1.135571	1.163392	1.188989	1.212441	1.233865	1.253339	1.270973	1.286847	1.301062
180.	60.	1.094188	1.138701	1.181491	1.222537	1.261857	1.299441	1.335317	1.369481	1.401977	1.432805	1.462005
180.	120.	1.087873	1.128657	1.167381	1.204052	1.238723	1.271411	1.302177	1.331047	1.358088	1.383327	1.406832
180.	180.	1.084209	1.122853	1.159259	1.193459	1.225520	1.255483	1.283424	1.309388	1.333458	1.355680	1.376131
180.	240.	1.082072	1.119460	1.154498	1.187234	1.217743	1.246080	1.272331	1.296552	1.318831	1.339231	1.357835
180.	300.	1.077855	1.112815	1.145250	1.175235	1.202872	1.228240	1.251447	1.272569	1.291718	1.308971	1.324432
240.	60.	1.098050	1.144382	1.188914	1.231624	1.272535	1.311630	1.348948	1.384478	1.418267	1.450318	1.480672
240.	120.	1.091750	1.134358	1.174829	1.213173	1.249439	1.283648	1.315858	1.346099	1.374437	1.400903	1.425562
240.	180.	1.085631	1.124678	1.161304	1.195552	1.227510	1.257229	1.284799	1.310278	1.333761	1.355306	1.375002
240.	240.	1.083517	1.121342	1.156649	1.189499	1.219989	1.248184	1.274183	1.298056	1.319906	1.339802	1.357840
240.	300.	1.082998	1.120480	1.155391	1.187793	1.217781	1.245424	1.270828	1.294062	1.315230	1.334408	1.351695
300.	60.	1.101751	1.149831	1.196045	1.240367	1.282822	1.323393	1.362117	1.398989	1.434052	1.467314	1.498812
300.	120.	1.095100	1.139251	1.181178	1.220893	1.258446	1.293860	1.327199	1.358490	1.387804	1.415175	1.440668
300.	180.	1.089209	1.129929	1.168150	1.203918	1.237318	1.268403	1.297265	1.323961	1.348589	1.371208	1.391908
300.	240.	1.083371	1.120746	1.155388	1.187383	1.216852	1.243881	1.268591	1.291071	1.311443	1.329794	1.346233
300.	300.	1.082891	1.119976	1.154299	1.185948	1.215045	1.241681	1.265979	1.288029	1.307956	1.325849	1.341822

Fig. 8-3 (A)

DECISION RULE 4C
F(N2)=((N2+.11)(M+T1+111)/(M+T2+111))-.11

TABLE OF VALUES FOR THE RATIO OF UNRELIABILITY TO MINIMUM SYSTEM UNRELIABILITY

LAMBDA2 = 0.00100
M = 60.00000

LAMBDA1 VALUES

T2	T1	0.0012	0.0013	0.0014	0.0015	0.0016	0.0017	0.0018	0.0019	0.0020	0.0021	0.0022
60.	60.	1.085843	1.126448	1.165502	1.202984	1.238912	1.273273	1.306093	1.337367	1.367134	1.395393	1.422179
60.	120.	1.151617	1.222075	1.289028	1.352493	1.412549	1.469226	1.522624	1.572781	1.619809	1.663761	1.704737
60.	180.	1.141674	1.206343	1.267039	1.323838	1.376876	1.426235	1.472063	1.514450	1.553551	1.589460	1.622322
60.	240.	1.132383	1.191723	1.246719	1.297509	1.344282	1.387177	1.426388	1.462051	1.494367	1.523467	1.549532
60.	300.	1.124145	1.178886	1.229053	1.274843	1.316498	1.354205	1.388205	1.418675	1.445847	1.469889	1.491005
120.	60.	1.01835	1.027027	1.035377	1.043389	1.051069	1.058414	1.065433	1.072119	1.078482	1.084524	1.090251
120.	120.	1.084979	1.124543	1.162188	1.197915	1.231770	1.263764	1.293951	1.322351	1.349020	1.373987	1.397305
120.	180.	1.143348	1.208982	1.270717	1.328614	1.382803	1.433355	1.480411	1.524050	1.564421	1.601612	1.635756
120.	240.	1.134466	1.194988	1.251244	1.303352	1.351491	1.395787	1.436422	1.473526	1.507284	1.537823	1.565314
120.	300.	1.126136	1.181936	1.233179	1.280044	1.322759	1.361501	1.396499	1.427921	1.455992	1.480874	1.502769
180.	60.	1.026785	1.039460	1.051658	1.063368	1.074597	1.085340	1.095609	1.105396	1.114717	1.123569	1.131964
180.	120.	1.025081	1.036748	1.047845	1.058371	1.068339	1.077754	1.086633	1.094980	1.102814	1.110142	1.116982
180.	180.	1.084209	1.122853	1.159259	1.193459	1.225520	1.255483	1.283424	1.309388	1.333458	1.355680	1.376131
180.	240.	1.136444	1.198094	1.255548	1.308917	1.358362	1.404001	1.446005	1.484491	1.519637	1.551564	1.580431
180.	300.	1.128444	1.185537	1.238141	1.286418	1.330582	1.370796	1.407274	1.440176	1.469713	1.496040	1.519351
240.	60.	1.034762	1.051223	1.067066	1.082284	1.096883	1.110854	1.124209	1.136949	1.149083	1.160612	1.171553
240.	120.	1.032618	1.047807	1.062262	1.075984	1.088991	1.101285	1.112887	1.123805	1.134061	1.143664	1.152637
240.	180.	1.030602	1.044616	1.057798	1.070162	1.081737	1.092535	1.102587	1.111912	1.120539	1.128488	1.135787
240.	240.	1.083517	1.121342	1.156649	1.189499	1.219989	1.248184	1.274183	1.298056	1.319906	1.339802	1.357840
240.	300.	1.130645	1.188977	1.242889	1.292526	1.338090	1.379727	1.417644	1.451986	1.482957	1.510702	1.535405
300.	60.	1.041920	1.061732	1.080778	1.099048	1.116549	1.133278	1.149246	1.164454	1.178917	1.192640	1.205638
300.	120.	1.039769	1.058309	1.075961	1.092731	1.108635	1.123679	1.137889	1.151267	1.163845	1.175632	1.186653
300.	180.	1.037391	1.054539	1.070686	1.085850	1.100059	1.113333	1.125705	1.137196	1.147842	1.157666	1.166703
300.	240.	1.035148	1.051003	1.065764	1.079460	1.092134	1.103818	1.114557	1.124385	1.133346	1.141472	1.148808
300.	300.	1.082891	1.119976	1.154299	1.185948	1.215045	1.241681	1.265979	1.288029	1.307956	1.325849	1.341822

F(N2)=N2*T1/T2

TABLE OF VALUES FOR THE RATIO OF M.L.E. UNRELIABILITY TO MINIMUM SYSTEM UNRELIABILITY

LAMBDA2 = 0.01000
M = 60.00000

LAMBDA1 VALUES

T2	T1	0.0010	0.0020	0.0030	0.0040	0.0050	0.0060	0.0070	0.0080	0.0090	0.0100	0.0110
60.	60.	1.484441	1.343804	1.252705	1.187678	1.138315	1.099267	1.067487	1.041102	1.018888	1.000000	1.015479
60.	120.	1.494485	1.356304	1.264938	1.198544	1.147373	1.106380	1.072664	1.044428	1.020481	1.000000	1.013984
60.	180.	1.515745	1.378596	1.283274	1.212232	1.156990	1.112780	1.076647	1.046640	1.021412	1.000000	1.013277
60.	240.	1.537070	1.400624	1.300792	1.224627	1.165076	1.117670	1.079354	1.047954	1.021888	1.000000	1.013006
60.	300.	1.557318	1.420693	1.316144	1.235027	1.171489	1.121260	1.081134	1.048689	1.022098	1.000000	1.012956
120.	60.	1.301765	1.234645	1.184776	1.144801	1.111390	1.082779	1.057920	1.036135	1.016945	1.000000	1.016708
120.	120.	1.306499	1.241563	1.192485	1.152415	1.118322	1.088637	1.062456	1.039203	1.018480	1.000000	1.015165
120.	180.	1.291457	1.227437	1.182516	1.146470	1.115407	1.087687	1.062570	1.039701	1.018886	1.000000	1.014558
120.	240.	1.302481	1.238438	1.191205	1.152768	1.119762	1.090585	1.064404	1.040754	1.019350	1.000000	1.014162
120.	300.	1.307961	1.239715	1.189949	1.150960	1.118378	1.089857	1.064205	1.040837	1.019480	1.000000	1.013915
180.	60.	1.202229	1.175572	1.148255	1.121889	1.097084	1.074064	1.052888	1.033532	1.015930	1.000000	1.017347
180.	120.	1.206450	1.182337	1.156251	1.130113	1.104788	1.080711	1.058114	1.037107	1.017735	1.000000	1.015516
180.	180.	1.201653	1.177886	1.153311	1.128682	1.104506	1.081142	1.058855	1.037828	1.018185	1.000000	1.014936
180.	240.	1.174397	1.150836	1.132831	1.115126	1.096500	1.077024	1.057158	1.037414	1.018244	1.000000	1.014545
180.	300.	1.185702	1.165985	1.146782	1.125930	1.103972	1.081729	1.059843	1.038753	1.018744	1.000000	1.014237
240.	60.	1.148373	1.144094	1.129109	1.110089	1.089857	1.069756	1.050458	1.032309	1.015468	1.000000	1.017617
240.	120.	1.126913	1.121296	1.110449	1.096570	1.080962	1.064493	1.047776	1.031257	1.015255	1.000000	1.017352
240.	180.	1.149503	1.147817	1.135072	1.117426	1.097620	1.077076	1.056616	1.036751	1.017808	1.000000	1.015892
240.	240.	1.136495	1.134799	1.125144	1.110842	1.093775	1.075182	1.055943	1.036704	1.017941	1.000000	1.014748
240.	300.	1.106703	1.104711	1.102003	1.095305	1.084470	1.070308	1.053864	1.036131	1.017955	1.000000	1.014372
300.	60.	1.119499	1.127656	1.119409	1.104315	1.086463	1.067827	1.049434	1.031830	1.015304	1.000000	1.017686
300.	120.	1.117595	1.127799	1.121838	1.108339	1.091230	1.072568	1.053529	1.034824	1.016891	1.000000	1.015965
300.	180.	1.097150	1.107582	1.106645	1.098464	1.085646	1.069988	1.052781	1.034962	1.017207	1.000000	1.015314
300.	240.	1.109743	1.119806	1.115916	1.104860	1.089847	1.072662	1.054431	1.035909	1.017635	1.000000	1.014908
300.	300.	1.094289	1.104012	1.103668	1.096611	1.084935	1.070157	1.053448	1.035730	1.017728	1.000000	1.014583

Fig. 8-8 (A)

DECISION RULE 4,A
F(N2)=(N2+1)(T1+M)/(T2+M)-1

TABLE OF VALUES FOR THE RATIO OF UNRELIABILITY TO MINIMUM SYSTEM UNRELIABILITY

LAMBDA2 = 0.01000
M = 60.00000

LAMBDA1 VALUES

T2	T1	0.0010	0.0020	0.0030	0.0040	0.0050	0.0060	0.0070	0.0080	0.0090	0.0100	0.0110
60.	60.	1.484441	1.343804	1.252705	1.187678	1.138315	1.099267	1.067487	1.041102	1.018888	1.000000	1.015479
60.	120.	1.101934	1.128311	1.126673	1.113249	1.094868	1.074759	1.054559	1.035113	1.016858	1.000000	1.016285
60.	180.	1.079510	1.103583	1.105408	1.096803	1.083052	1.066856	1.049727	1.032552	1.015866	1.000000	1.016833
60.	240.	1.021791	1.049097	1.065390	1.070454	1.067013	1.057844	1.045167	1.030605	1.015294	1.000000	1.016913
60.	300.	1.017945	1.042108	1.057873	1.063908	1.061978	1.054338	1.042970	1.029423	1.014833	1.000000	1.017163
120.	60.	1.072971	1.093474	1.093789	1.085117	1.072298	1.057710	1.042619	1.027732	1.013451	1.000000	1.019261
120.	120.	1.306499	1.241563	1.192485	1.152415	1.118322	1.088637	1.062456	1.039203	1.018480	1.000000	1.015165
120.	180.	1.088566	1.116457	1.119126	1.109633	1.094040	1.075557	1.056019	1.036520	1.017717	1.000000	1.015222
120.	240.	1.104801	1.126401	1.121792	1.107777	1.090274	1.071619	1.052844	1.034466	1.016786	1.000000	1.015904
120.	300.	1.073058	1.096264	1.099473	1.092939	1.081155	1.066455	1.050222	1.033354	1.016467	1.000000	1.015922
180.	60.	1.302618	1.237400	1.187981	1.147833	1.113980	1.084825	1.059402	1.037074	1.017387	1.000000	1.016311
180.	120.	1.088215	1.113665	1.114599	1.104409	1.088953	1.071163	1.052633	1.034278	1.016629	1.000000	1.016231
180.	180.	1.201653	1.177886	1.153311	1.128682	1.104506	1.081142	1.058855	1.037828	1.018185	1.000000	1.014936
180.	240.	1.069533	1.096817	1.103772	1.099223	1.087813	1.072380	1.054794	1.036333	1.017870	1.000000	1.014734
180.	300.	1.078945	1.102551	1.104817	1.097207	1.084503	1.069038	1.052129	1.034617	1.017097	1.000000	1.015291
240.	60.	1.079446	1.101536	1.101650	1.092050	1.078019	1.062145	1.045800	1.029742	1.014398	1.000000	1.018406
240.	120.	1.106875	1.135708	1.134988	1.121461	1.102298	1.080975	1.059306	1.038276	1.018414	1.000000	1.014780
240.	180.	1.088392	1.114390	1.115978	1.106331	1.091184	1.073421	1.054642	1.035791	1.017454	1.000000	1.015294
240.	240.	1.136495	1.134799	1.125144	1.110842	1.093775	1.075182	1.055943	1.036704	1.017941	1.000000	1.014748
240.	300.	1.052504	1.078417	1.088682	1.088440	1.080937	1.068529	1.053011	1.035765	1.017835	1.000000	1.014426
300.	60.	1.251826	1.209230	1.171905	1.138676	1.108906	1.082170	1.058153	1.036595	1.017275	1.000000	1.016264
300.	120.	1.208043	1.191484	1.167207	1.140478	1.113564	1.087550	1.062986	1.040144	1.019139	1.000000	1.014304
300.	180.	1.090778	1.118956	1.121628	1.112103	1.096416	1.077725	1.057836	1.037842	1.018422	1.000000	1.014444
300.	240.	1.078157	1.103536	1.107164	1.100070	1.087230	1.071263	1.053719	1.035583	1.017523	1.000000	1.014965
300.	300.	1.094289	1.104012	1.103668	1.096611	1.084935	1.070157	1.053448	1.035730	1.017728	1.000000	1.014583

Fig. 8-9(A)

TABLE OF VALUES FOR THE RATIO OF UNRELIABILITY TO MINIMUM SYSTEM UNRELIABILITY

LAMBDA2 = 0.01000
M = 60.00000

LAMBDA1 VALUES

T2	T1	0.0010	0.0020	0.0030	0.0040	0.0050	0.0060	0.0070	0.0080	0.0090	0.0100	0.0110
60.	60.	1.484441	1.343804	1.252705	1.187678	1.138315	1.099267	1.067487	1.041102	1.018888	1.000000	1.015479
60.	120.	1.496202	1.360345	1.270526	1.204761	1.153444	1.111730	1.076908	1.047334	1.021942	1.000000	1.012528
60.	180.	1.519557	1.387431	1.295287	1.225353	1.169557	1.123634	1.085079	1.052294	1.024194	1.000000	1.010621
60.	240.	1.538062	1.404838	1.308678	1.235180	1.176699	1.128766	1.088629	1.054516	1.025246	1.000000	1.009645
60.	300.	1.558340	1.425223	1.324914	1.247071	1.185014	1.134351	1.092174	1.056538	1.026120	1.000000	1.008953
120.	60.	1.291729	1.222074	1.172446	1.133865	1.102316	1.075705	1.052820	1.032895	1.015411	1.000000	1.018107
120.	120.	1.306499	1.241563	1.192485	1.152415	1.118322	1.088637	1.062456	1.039203	1.018480	1.000000	1.015165
120.	180.	1.291601	1.228080	1.183779	1.148240	1.117446	1.089721	1.064344	1.041010	1.019583	1.000000	1.013806
120.	240.	1.309103	1.252208	1.208128	1.169597	1.134540	1.102367	1.072908	1.046085	1.021817	1.000000	1.012047
120.	300.	1.317725	1.258953	1.212481	1.172450	1.136601	1.103991	1.074195	1.047009	1.022309	1.000000	1.011503
180.	60.	1.182522	1.151043	1.124341	1.100806	1.079692	1.060583	1.043222	1.027427	1.013057	1.000000	1.019942
180.	120.	1.205607	1.180345	1.153484	1.127025	1.101765	1.078042	1.055994	1.035654	1.017005	1.000000	1.016241
180.	180.	1.201653	1.177886	1.153311	1.128682	1.104506	1.081142	1.058855	1.037828	1.018185	1.000000	1.014936
180.	240.	1.174412	1.150957	1.133167	1.115720	1.097309	1.077939	1.058036	1.038112	1.018639	1.000000	1.014083
180.	300.	1.186424	1.168811	1.151684	1.132042	1.110279	1.087398	1.064325	1.041765	1.020214	1.000000	1.012887
240.	60.	1.123434	1.113117	1.098977	1.083582	1.068037	1.052878	1.038383	1.024697	1.011893	1.000000	1.020832
240.	120.	1.126255	1.119737	1.108279	1.094145	1.078583	1.062387	1.046101	1.030106	1.014675	1.000000	1.017931
240.	180.	1.149418	1.147442	1.134334	1.116389	1.096422	1.075878	1.055569	1.035976	1.017395	1.000000	1.015538
240.	240.	1.136495	1.134799	1.125144	1.110842	1.093775	1.075182	1.055943	1.036704	1.017941	1.000000	1.014748
240.	300.	1.106704	1.104735	1.102100	1.095521	1.084816	1.070751	1.054332	1.036533	1.018195	1.000000	1.014066
300.	60.	1.089222	1.090115	1.082953	1.072300	1.060153	1.047512	1.034924	1.022698	1.011022	1.000000	1.021524
300.	120.	1.096960	1.102155	1.096940	1.086530	1.073392	1.058886	1.043839	1.028789	1.014094	1.000000	1.018404
300.	180.	1.094638	1.101932	1.099174	1.090523	1.078236	1.063749	1.048056	1.031871	1.015723	1.000000	1.016661
300.	240.	1.109734	1.119730	1.115704	1.104484	1.089334	1.072082	1.053872	1.035463	1.017384	1.000000	1.015204
300.	300.	1.094289	1.104012	1.103668	1.096611	1.084935	1.070157	1.053448	1.035730	1.017728	1.000000	1.014583

Fig. 8-10(A)

DECISION RULE 4,A
F(N2)=(N2+1)(T1+M)/(T2+M)-1

TABLE OF VALUES FOR THE RATIO OF UNRELIABILITY TO MINIMUM SYSTEM UNRELIABILITY

LAMBDA2 = 0.01000
M = 60.00000

LAMBDA1 VALUES

T2	T1	0.0010	0.0020	0.0030	0.0040	0.0050	0.0060	0.0070	0.0080	0.0090	0.0100	0.0110
60.	60.	1.484441	1.343804	1.252705	1.187678	1.138315	1.099267	1.067487	1.041102	1.018888	1.000000	1.015479
60.	120.	1.101934	1.128311	1.126673	1.113249	1.094868	1.074759	1.054559	1.035113	1.016858	1.000000	1.016285
60.	180.	1.079510	1.103583	1.105408	1.096803	1.083052	1.066856	1.049727	1.032552	1.015866	1.000000	1.016833
60.	240.	1.021791	1.049097	1.065390	1.070454	1.067013	1.057844	1.045167	1.030605	1.015294	1.000000	1.016913
60.	300.	1.017945	1.042108	1.057873	1.063908	1.061978	1.054338	1.042970	1.029423	1.014833	1.000000	1.017163
120.	60.	1.072971	1.093474	1.093789	1.085117	1.072298	1.057710	1.042619	1.027732	1.013451	1.000000	1.019261
120.	120.	1.306499	1.241563	1.192485	1.152415	1.118322	1.088637	1.062456	1.039203	1.018480	1.000000	1.015165
120.	180.	1.088566	1.116457	1.119126	1.109633	1.094040	1.075557	1.056019	1.036520	1.017717	1.000000	1.015222
120.	240.	1.104801	1.126401	1.121792	1.107777	1.090274	1.071619	1.052844	1.034466	1.016786	1.000000	1.015904
120.	300.	1.073058	1.096264	1.099473	1.092939	1.081155	1.066455	1.050222	1.033354	1.016467	1.000000	1.015922
180.	60.	1.302618	1.237400	1.187981	1.147833	1.113980	1.084825	1.059402	1.037074	1.017387	1.000000	1.016311
180.	120.	1.088215	1.113665	1.114599	1.104409	1.088953	1.071163	1.052633	1.034278	1.016629	1.000000	1.016231
180.	180.	1.201653	1.177886	1.153311	1.128682	1.104506	1.081142	1.058855	1.037828	1.018185	1.000000	1.014936
180.	240.	1.069533	1.096817	1.103772	1.099223	1.087813	1.072380	1.054794	1.036333	1.017870	1.000000	1.014734
180.	300.	1.078945	1.102551	1.104817	1.097207	1.084503	1.069038	1.052129	1.034617	1.017097	1.000000	1.015291
240.	60.	1.079446	1.101536	1.101650	1.092050	1.078019	1.062145	1.045800	1.029742	1.014398	1.000000	1.018406
240.	120.	1.106875	1.135708	1.134988	1.121461	1.102298	1.080975	1.059306	1.038276	1.018414	1.000000	1.014780
240.	180.	1.088392	1.114390	1.115978	1.106331	1.091184	1.073421	1.054642	1.035791	1.017454	1.000000	1.015294
240.	240.	1.136495	1.134799	1.125144	1.110842	1.093775	1.075182	1.055943	1.036704	1.017941	1.000000	1.014748
240.	300.	1.052504	1.078417	1.088682	1.088440	1.080937	1.068529	1.053011	1.035765	1.017835	1.000000	1.014426
300.	60.	1.251826	1.209230	1.171905	1.138676	1.108906	1.082170	1.058153	1.036595	1.017275	1.000000	1.016264
300.	120.	1.208043	1.191484	1.167207	1.140478	1.113564	1.087550	1.062986	1.040144	1.019139	1.000000	1.014304
300.	180.	1.090778	1.118956	1.121628	1.112103	1.096416	1.077725	1.057836	1.037842	1.018422	1.000000	1.014444
300.	240.	1.078157	1.103536	1.107164	1.100070	1.087230	1.071263	1.053719	1.035583	1.017523	1.000000	1.014965
300.	300.	1.094289	1.104012	1.103668	1.096611	1.084935	1.070157	1.053448	1.035730	1.017728	1.000000	1.014583

Fig. 8-9(A)

TABLE OF VALUES FOR THE RATIO OF UNRELIABILITY TO MINIMUM SYSTEM UNRELIABILITY

LAMBDA2 = 0.01000
M = 60.00000

LAMBDA1 VALUES

T2	T1	0.0010	0.0020	0.0030	0.0040	0.0050	0.0060	0.0070	0.0080	0.0090	0.0100	0.0110
60.	60.	1.484441	1.343804	1.252705	1.187678	1.138315	1.099267	1.067487	1.041102	1.018888	1.000000	1.015479
60.	120.	1.496202	1.360345	1.270526	1.204761	1.153444	1.111730	1.076908	1.047334	1.021942	1.000000	1.012528
60.	180.	1.519557	1.387431	1.295287	1.225353	1.169557	1.123634	1.085079	1.052294	1.024194	1.000000	1.010621
60.	240.	1.538062	1.404838	1.308678	1.235180	1.176699	1.128766	1.088629	1.054516	1.025246	1.000000	1.009645
60.	300.	1.558340	1.425223	1.324914	1.247071	1.185014	1.134351	1.092174	1.056538	1.026120	1.000000	1.008953
120.	60.	1.291729	1.222074	1.172446	1.133865	1.102316	1.075705	1.052820	1.032895	1.015411	1.000000	1.018107
120.	120.	1.306499	1.241563	1.192485	1.152415	1.118322	1.088637	1.062456	1.039203	1.018480	1.000000	1.015165
120.	180.	1.291601	1.228080	1.183779	1.148240	1.117446	1.089721	1.064344	1.041010	1.019583	1.000000	1.013806
120.	240.	1.309103	1.252208	1.208128	1.169597	1.134540	1.102367	1.072908	1.046085	1.021817	1.000000	1.012047
120.	300.	1.317725	1.258953	1.212481	1.172450	1.136601	1.103991	1.074195	1.047009	1.022309	1.000000	1.011503
180.	60.	1.182522	1.151043	1.124341	1.100806	1.079692	1.060583	1.043222	1.027427	1.013057	1.000000	1.019942
180.	120.	1.205607	1.180345	1.153484	1.127025	1.101765	1.078042	1.055994	1.035654	1.017005	1.000000	1.016241
180.	180.	1.201653	1.177886	1.153311	1.128682	1.104506	1.081142	1.058855	1.037828	1.018185	1.000000	1.014936
180.	240.	1.174412	1.150957	1.133167	1.115720	1.097309	1.077939	1.058036	1.038112	1.018639	1.000000	1.014083
180.	300.	1.186424	1.168811	1.151684	1.132042	1.110279	1.087398	1.064325	1.041765	1.020214	1.000000	1.012887
240.	60.	1.123434	1.113117	1.098977	1.083582	1.068037	1.052878	1.038383	1.024697	1.011893	1.000000	1.020832
240.	120.	1.126255	1.119737	1.108279	1.094145	1.078583	1.062387	1.046101	1.030106	1.014675	1.000000	1.017931
240.	180.	1.149418	1.147442	1.134334	1.116389	1.096422	1.075878	1.055569	1.035976	1.017395	1.000000	1.015538
240.	240.	1.136495	1.134799	1.125144	1.110842	1.093775	1.075182	1.055943	1.036704	1.017941	1.000000	1.014748
240.	300.	1.106704	1.104735	1.102100	1.095521	1.084816	1.070751	1.054332	1.036533	1.018195	1.000000	1.014066
300.	60.	1.089222	1.090115	1.082953	1.072300	1.060153	1.047512	1.034924	1.022698	1.011022	1.000000	1.021524
300.	120.	1.096560	1.102155	1.096940	1.086530	1.073392	1.058886	1.043839	1.028789	1.014094	1.000000	1.018404
300.	180.	1.094638	1.101932	1.099174	1.090523	1.078236	1.063749	1.048056	1.031871	1.015723	1.000000	1.016661
300.	240.	1.109734	1.119730	1.115704	1.104484	1.089334	1.072082	1.053872	1.035463	1.017384	1.000000	1.015204
300.	300.	1.094289	1.104012	1.103668	1.096611	1.084935	1.070157	1.053448	1.035730	1.017728	1.000000	1.014583

Fig. 8-10(A)

DECISION RUL' 4,C
F(N2)=((N2+.11)(M+T1+111)/(M+T2+111))-.11

TABLE OF VALUES FOR THE RATIO OF UNRELIABILITY TO MINIMUM SYSTEM UNRELIABILITY

LAMBDA2 = 0.01000
M = 60.00000

LAMBDA1 VALUES

T2	T1	0.0010	0.0020	0.0030	0.0040	0.0050	0.0060	0.0070	0.0080	0.0090	0.0100	0.0110
60.	60.	1.484441	1.343804	1.252705	1.187678	1.138315	1.099267	1.067487	1.041102	1.018888	1.000000	1.015479
60.	120.	1.103671	1.132442	1.132443	1.119728	1.101247	1.080424	1.059082	1.038230	1.018433	1.000000	1.014703
60.	180.	1.153462	1.193103	1.189965	1.168813	1.140288	1.109510	1.079076	1.050313	1.023867	1.000000	1.010241
60.	240.	1.201951	1.250430	1.242627	1.212318	1.173791	1.133687	1.095187	1.059762	1.027994	1.000000	1.007031
60.	300.	1.231341	1.269629	1.250359	1.213058	1.171412	1.130579	1.092552	1.058043	1.027219	1.000000	1.007591
120.	60.	1.520368	1.362857	1.262865	1.192881	1.140717	1.100131	1.067583	1.040909	1.018700	1.000000	1.015781
120.	120.	1.306499	1.241563	1.192485	1.152415	1.118322	1.088637	1.062456	1.039203	1.018480	1.000000	1.015165
120.	180.	1.088713	1.117125	1.120458	1.111531	1.096264	1.077812	1.058019	1.038018	1.018527	1.000000	1.014322
120.	240.	1.118662	1.156397	1.159950	1.146834	1.125381	1.100113	1.073665	1.047609	1.022880	1.000000	1.010731
120.	300.	1.147540	1.190884	1.190755	1.171070	1.142976	1.112055	1.081174	1.051798	1.024638	1.000000	1.009418
180.	60.	1.295764	1.213166	1.159294	1.120337	1.090219	1.065858	1.045527	1.028181	1.013155	1.000000	1.020192
180.	120.	1.323826	1.248945	1.194983	1.152500	1.117318	1.087293	1.061196	1.038265	1.017987	1.000000	1.015678
180.	180.	1.201653	1.177886	1.153311	1.128682	1.104506	1.081142	1.058855	1.037828	1.018185	1.000000	1.014936
180.	240.	1.069548	1.096943	1.104132	1.099870	1.088712	1.073417	1.055811	1.037157	1.018345	1.000000	1.014155
180.	300.	1.088387	1.123695	1.132198	1.125394	1.109759	1.089340	1.066753	1.043691	1.021222	1.000000	1.011930
240.	60.	1.168096	1.125181	1.096432	1.074925	1.057652	1.043109	1.030471	1.019255	1.009162	1.000000	1.023533
240.	120.	1.188978	1.152363	1.124186	1.100573	1.079832	1.061121	1.043990	1.028184	1.013551	1.000000	1.019233
240.	180.	1.210526	1.180852	1.153378	1.127393	1.102725	1.079370	1.057377	1.036796	1.017663	1.000000	1.015450
240.	240.	1.136495	1.134799	1.125144	1.110842	1.093775	1.075182	1.055943	1.036704	1.017941	1.000000	1.014748
240.	300.	1.052505	1.078443	1.088787	1.088677	1.081326	1.069039	1.053563	1.036248	1.018131	1.000000	1.014031
300.	60.	1.116479	1.099983	1.084557	1.070049	1.056398	1.043566	1.031530	1.020270	1.009767	1.000000	1.022820
300.	120.	1.112424	1.398245	1.085916	1.073801	1.061494	1.048993	1.036431	1.023975	1.011785	1.000000	1.020534
300.	180.	1.125730	1.114913	1.102736	1.089432	1.075218	1.060355	1.045128	1.029822	1.014702	1.000000	1.017622
300.	240.	1.141317	1.135997	1.124440	1.109237	1.091886	1.073393	1.054484	1.035698	1.017438	1.000000	1.015232
300.	300.	1.094289	1.104012	1.103668	1.096611	1.084935	1.070157	1.053448	1.035730	1.017728	1.000000	1.014583

Fig. 8-11 (A)

TABLE OF VALUES FOR RATIO OF UNRELIABILITY USING DECISION RULE TO MINIMUM SYSTEM UNRELIABILITY

LAMBDA2 = 0.001000

M = 60.000000

LAMBDA1 VALUES

N	0.001200	0.001300	0.001400	0.001500	0.001600	0.001700	0.001800	0.001900	0.002000	0.002100	0.002200
1.00	1.085847	1.126451	1.165511	1.202999	1.238935	1.273304	1.306138	1.337426	1.367208	1.395486	1.422293
2.00	1.084588	1.124560	1.162219	1.197967	1.231849	1.263877	1.294104	1.322548	1.349271	1.374297	1.397683
3.00	1.084225	1.122888	1.159324	1.193562	1.225674	1.255700	1.283717	1.309767	1.333935	1.356268	1.376842
4.00	1.083541	1.121395	1.156748	1.189660	1.220229	1.248519	1.274631	1.298634	1.320630	1.340691	1.358908
5.00	1.082922	1.120050	1.154436	1.186170	1.215373	1.242137	1.266584	1.288807	1.308927	1.327035	1.343242
6.00	1.082358	1.118828	1.152341	1.183015	1.210998	1.236403	1.259375	1.280028	1.298502	1.314995	1.329366

DECISION RULE 2

Fig. 8 - 15(A)

TABLE OF VALUES FOR RATIO OF UNRELIABILITY USING DECISION RULE TO MINIMUM SYSTEM UNRELIABILITY

LAMBDA2 = 0.010000

M = 60.000000

N	LAMBDA1 VALUES										
	0.001000	0.002000	0.003000	0.004000	0.005000	0.006000	0.007000	0.008000	0.009000	0.010000	0.011000
1.00	1.489565	1.349565	1.257697	1.191512	1.140994	1.100955	1.068410	1.041498	1.018982	1.000000	1.015568
2.00	1.317446	1.253537	1.202583	1.159962	1.123454	1.091786	1.064132	1.039901	1.018643	1.000000	1.015310
3.00	1.215356	1.193027	1.166173	1.138337	1.111085	1.085180	1.061000	1.038719	1.018391	1.000000	1.015117
4.00	1.150558	1.150947	1.139264	1.121675	1.101274	1.079834	1.058431	1.037741	1.018181	1.000000	1.014956
5.00	1.107397	1.119962	1.118207	1.108104	1.093064	1.075276	1.056214	1.036890	1.017998	1.000000	1.014816
6.00	1.077678	1.096360	1.101191	1.096706	1.085987	1.071276	1.054244	1.036129	1.017833	1.000000	1.014690

Fig. 8-16(A)

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13. ABSTRACT

Specific decision rules for the spare parts allocation procedure for a system composed of two failure prone items are formulated and analysed. It is considered desirable to have the system operate successfully during a mission when space for only one spare is available. The decision rules formulated consider the use of operational data to decide where a spare is best placed. These rules have been numerically evaluated for several specific exponential time to failure distributions and tables associated with this evaluation are included.

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